PERMEABILITY AND WATER FLOW IN SOILS AND SEDIMENTS

by

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For my father and mother,

may Allah be pleased with them.

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University regulations under which this thesis is submitted.

This thesis is submitted as "a critical review of existing knowledge" under Glasgow University regulations for M.Sc.'s. (University of Glasgow calendar 1988-89 (1988) p.429-430).

The following represent original work by me in the thesis:

1) Page 14, the derivation of Darcy's law.
2) Page 31, the derivation of Hooghoudt's (1936) equation.
3) Page 49, I have equated Smith's (1981) and Capper and Cassie's (1976) equations using the derivation of them for vertical and horizontal flow.
4) Page 109, I have equated equation (4) and (5) of the projection sphericity.
5) Page 207, I have calculated the regression lines of Wallace's data using the Minitab statistical package.
6) Page 142, I have developed the relationship between water content, degree of saturation and void ratio.

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INTRODUCTION

The general topic of flow through porous media covers a wide field ranging from industrial processes in factories to the movement of oil or gas in oil-fields, and also the movement of water in soils and sediments. In my thesis I am concerned the flow of water into, and through, soils and sediments.

Water is one of the most powerful forces of nature. It moves through, or is held in or drawn into, the pores of soils and sediments. It has significance in many types of hydrological, agricultural, and geological problems (Lambe, 1955; Taylor and Ashcroft, 1972; Capper and Cassie, 1976; Cedergren, 1977).

The measurement of the speed at which water flows through soils and sediments is called permeability or hydraulic conductivity. This is one of the most important properties of soils and sediments, and can be measured in the laboratory and in the field (Fraser, 1935; Hooghoudt, 1936; Kirkham, 1946; Frevert and Kirkham, 1948; Luthin and Kirkham, 1949; Smiles and Youngs, 1965; Childs and Collis-George, 1950; Terzaghi and Peck, 1967; Stakman, 1972; Scott, 1974; Dunn et al. 1980; Smith, 1981; Cernica, 1982; Das, 1985). Permeability plays a vital part in problems related to drainage, wells, groundwater storage, agricultural lands, railroads, buildings and seepage through earth dams and levees (Lambe, 1955; Hillel, 1971; Hulings and Gray, 1971; Scott, 1974; Bowles, 1979; Smith, 1981).

There are many examples showing that permeability plays a vital role in these problems. In drainage for instance, the pore spaces must be large enough to impart sufficient permeability to permit water to escape freely and thus provide a high degree of control over seepage

forces and hydrostatic pressures (Cedergren, 1977).

Embankments for major railroads will collapse if subsurface drainage is ineffective and does not lower the ground water level (Luthin, 1966; Terzaghi and Peck, 1967; Cedergren, 1977).

Another example is that of plants wilting when the available water has been drawn out of the soil because of its high permeability. Any water left in the soil is hygroscopic and held too strongly to be extracted by roots. The wilting point of plants depends on the species of plant and which environment it is adapted to. The following are four examples of species adapted to conditions in which the soil permeabilites and water content are very different.

I) Desert plants such as Calotropis procera and Rhazya stricta. These plants grow in Saudi Arabia (Migahid, 1978).

II) Plants of the desert and salt marsh such as Halopeplis perfolia and Zygophyllum coccineum (Migahid, 1978) these also grow in Saudi Arabia.

III) Temperate zone plants such as Peucedanum palustre and Peucedanum ostruthium. These plants grow in Britain (Butcher, 1961).

IV) Maritime plants such as Cakile maritima and Crambe maritime (Butcher, 1961). These also grow in Britain.

In building construction, permeability tests play a vital part in determining the bearing capacity of soils. If permeability is high, the bearing capacity of the soil will be low - and vice versa (Capper, et al. 1966; Zeevaert, 1972; Scott, 1974; Dunn, et al. 1980; Cernica, 1982; Lee, et al. 1983).

The testing engineer must be constantly aware of the different

properties that can affect the results obtained from permeability tests (Krumbein and Monk, 1942; Wallace, 1948; Scott, 1974; Wilun and Starzewski, 1975; Lambe and Whitman, 1979; Nowell et al. 1981; Cernica, 1982; Weaver and Schulteiss, 1983; Das, 1985). These various properties are particle size, particle shape, packing, void ratio, degree of saturation, fabric, composition and biological effects. For example, void ratio has a large effect on permeability because when a soil sample is compressed or vibrated, the volume occupied by its solid constituents remains almost unchanged, but the volume of the voids decreases. This causes in a decrease in soil permeability (Taylor, 1948; Wallace, 1948; Terzaghi and Peck, 1967; Lambe and Whitman, 1979). Another example shows the effect of degree of saturation on permeability (Wallace 1948). Wallace found that the higher the degree of saturation, the higher the permeability. This effect has also been reported by Lambe (1955), Lambe and Whitman (1979), and Das (1985). Meadows and Tufail (1986) have shown that micro-organisms reduce permeability significantly. This effect has also been reported by Plummer et al. (1944), Alison (1947), McCalla (1950), Webb (1969), Jenneman et al. (1984), and Shaw et al. (1985). Permeability is also affected by burrowing invertebrates whose burrows may increase sediment permeability (Smith et al. 1944; Nowell et al. 1981; Weaver and Schulteiss, 1983; Meadows and Tait, 1989).

The empirical law discovered in 1856 by the French hydraulic engineer Henri Darcy provides the basic mathematical equation for the study of water flow through soils and sediments. The rate of flow of water will depend on the magnitude of the forces and gradients and also on the factors determining the hydraulic conductivity of the soil. Water is held in the soil against gravitational forces draining

water out or against evaporation of water from the surface of the soil. The energy with which water is held in a soil at any water content can be specified as the water potential or soil-water potential. The potential energy of soil water varies over a very wide range. Differences in potential energy of water between one point and another give rise to the tendency of water to flow within the soil. The spontaneous and universal tendency of all matter in nature is to move from where the potential energy is high to where it is lower, and for each parcel of matter to equilibrate with its surroundings. Soil water obeys this equilibrium. It moves in the direction of decreasing potential energy (Harr, 1966; Hillel, 1971; Taylor and Ashcroft, 1972; Yong and Warkentin, 1975).

Section one and three (pp.13 and 181 respectively) both deal with Darcy's equation. This was felt necessary because Darcy's equation occurs in two different contexts. Section one gives the general equation of Darcy's law, its derivation and its dimensional analysis. In section three, Darcy's law is given an alternative treatment in which the quantity per unit area per unit time, Q/At, is called the flux density, J_w.

Throughout the thesis, soil and sediment are used in various contexts. Their use is almost synonymous unless where stated, because the flow of water through both is governed by the same laws, and is treated mathematically in an identical way. Soils are found in land, and sediments in fresh water or marine environments.

I have included a significant amount of mathematics in the thesis, because this is where my interests lie. With this background, the aim of this dissertation is as follows.

SECTION I gives an account of Darcy's Law and its derivation. It also describes the measurement of permeability of soils and sediments in the field and in the laboratory. The field methods are divided into those where the water table is present and those where the water table is absent.

SECTION II gives an account of the sediment properties influencing permeability such as particle size and shape, packing, void ratio, composition, fabric, and biological effects.

SECTION III explores the effect of water potentials and steady state flow in a horizontal and vertical direction through soils and sediments, and gives an account of the mathematical equations relating to those potentials and to the resultant water flow.

SECTION I Permeability and its measurements.

(1) Permeability is defined as the property of a soil which allows the seepage of fluids through its interconnected void spaces.

(2) The flow of water through soils is assumed to follow Darcy's law:

$$\frac{Q}{t} = k A \frac{H}{L}$$

where,

k = the coefficient of permeability

Q = the quantity of water flowing through the soil in time t

L = the height of the soil sample

A = the area of the cross-section through which the water flows

H = the constant head of water (hydraulic head) operating over the height of the soil.

(3) **Laboratory** measurements. The two most common laboratory methods for determining the coefficient of permeability of soils are the following:

i) **Constant head permeameter.** In the constant head test the level of the water is kept constant by addition of water.

ii) **Variable head permeameter.** In this test the level of the water does not remain constant because no water is added to the cylinder containing the sediment.

(4) **Field measurements.** Soil permeability in the field can be measured when the water table is present and when its absent.

4.1 **Water table present.** Several methods are presently available for the determination of the coefficient of permeability in

the field when the water table is present. In my thesis I discussed some of these methods, which are as follows:

4.1.1 Auger hole methods. The auger hole methods are as follows:

Hooghoudt's method. Hooghoudt (1936) mathematically analysed the auger hole method in a homogeneous soil based on his own experimental observations. Hooghoudt's equation is:

$$k = \frac{rs}{(2H+r)t} \ln \frac{y_0}{y}$$

where,

k = the coefficient of permeability

r = the radius of the auger hole

$$s = \frac{rH}{0.19}$$

H = the distance from the bottom of the hole to the water table.

The two auger hole method. Childs (1952) and Childs, et al. (1953) have described a method for non-layered soil using two auger holes. They consider two methods, one reaches an impermeable layer and the other does not.

The pipe cavity method. Kirkham (1946) described the pipe cavity method which consists of pushing a pipe into an auger hole slightly smaller in diameter than the pipe, using a special technique designed to eliminate compaction.

* y and y_o = vertical heights between water table in soil and water level in auger hole at times t and t_o.

Ernst's equation. Ernst (1950) developed an equation which can be used to measure permeability by the auger hole method. The equation was used for homogeneous soil with an impermeable layer at a great depth below the bottom of the auger hole. Here, the permeability coefficient is given by:

$$k = \frac{4000}{(20 + \frac{d}{a})(2 - \frac{y}{d})} \cdot \frac{a}{y} \cdot \frac{\Delta y}{\Delta t}$$

where,

k = coefficient of permeability

Δy = rise of water surface in auger hole during the time interval Δt

d = depth of water in hole before pumping

y = distance from static water table to elevation of water in the hole

a = radius of auger hole.

4.1.2 **Well and pumping method.** The determination of the coefficient of permeability is made when water flows through a surface area of $2\pi rH$ and when it flows through $2\pi d$.

4.3.1 **Multi-layer method.** Smith (1981) and Capper and Cassie (1976) each give two equations to determine the permeability of different layers of sediment, one for horizontal flow and the other for vertical flow.

4.2 **Water table absent.** Several methods are also available for the determination of the coefficient of permeability in the field

when the water table is absent. Some of these methods are as follows:

4.2.1 **Particle size method.** The coefficient for clean granular soil can be estimated from the following equation:

$$k = \frac{2g}{C_s} \frac{\rho}{\mu} D^2 \frac{e^3}{1+e}$$

where,

g = acceleration of gravity (9.81 m/sec^2)

$\frac{\mu}{\rho}$ = kinematic viscosity of water

Cs = particle shape factor

D = weight or characteristic particle diameter

e = void ratio.

4.2.2 **Shallow water pumping test.** The volume of water flowing horizontally from a well is measured. In this method a constant head of water is maintained by a float valve. The horizontal permeability obtained is a composite rate for the full depth of the hole being tested, but reflects primarily the permeability of the more permeable layers.

4.2.3 **The permeameter method.** In this method the flow can be calculated by an application of Darcy's law.

4.2.4 **Pond-Infiltration test.** This test, which is an infiltration test over a large area, has been recommended and put into practice to avoid the problem of soil compression which is inherent in core samples.

SECTION II Sediment properties influencing permeability.

Permeability depends on the characteristics of the soil which are as follows:

1 **Particle size.** An increase in grain size results an increase in permeability. Two methods for determining the particle size parameters are outlined

(I) Graphical analysis.

(II) Algebraic analysis.

2 **Particle shape.** The coefficient of permeability decreases with increasing uniformity of the pore spaces. The effect of particle shape on porosity is discussed using Fraser's experiment.

3 **Packing.** Permeability is dependent on the packing arrangement, because the tighter the packing density of particles in the soil or sediment the lower its effective porosity and hence the lower its permeability.

4 **Void ratio.** The void ratio of soils and sediments has an important effect on permeability. When the volume of voids decreases, the permeability also decreases.

5 **Composition.** Soil composition is of limited importance in the permeability of some soil types such as silts, sands and gravels, but it is of major importance in clays.

6 **Fabric.** Fabric is one of the most important sediment properties influencing permeability, especially in fine-grained soils. Soil samples which are in a flocculated state will have a higher permeability, while the ones in a more dispersed state will have a lower permeability.

7 Degree of saturation. The higher the degree of saturation, the higher the permeability. The relationship between degree of saturation, void ratio and water content have been algebraically solved and illustrated.

8 Biological effects. I have quoted some examples of these effects, such as the effects of intertidal burrowing invertebrates and micro-organisms.

SECTION III Soil physics.

1 Water potential. Water is held in the soil against gravitational forces draining water out, or against evaporation of water from the surface of the soil. The energy with which water is held in a soil at any water content is called the water potential. Water flows from where the potential energy is high to where it is low. It moves constantly in the direction of decreasing potential energy. Water potential consists of pressure potential, solute potential and matric potential. The sum of the gravitational potential and the water potential gives the total water potential.

2 Steady state flow in horizontal and vertical direction. In steady state flow, flow characteristics do not change with time, although they may change with location. The rate of flow will depend on the rate of decrease of potential energy in a horizontal and vertical direction. The general equation for horizontal flow is:

$$Jw = - k \frac{\Delta \psi_h}{\Delta s}$$

in which

Jw = water flux density (cm/s)

k = permeability coefficient (cm/s)

$\Delta \psi_h$ = the difference in hydraulic potential (cm) between two points separated by a distance Δs (cm) where s is a horizontal distance measured along the direction of flow.

For vertical flow the equation is

$$Jw = - k \frac{\Delta \psi_h}{\Delta z}$$

where

z = the distance (cm) measured in the vertical direction.

S E C T I O N O N E

PERMEABILITY

INTRODUCTION

Permeability is a measure of the speed at which water flows through soil and sediment, and it is important to the civil engineer who studies seepage under dams, ground water lowering and land drainage (Smith, 1981).

Permeability varies between different sediment types (Jumikis, 1962; Hansen, et al. 1980) and it depends on the size and shape of the particles, the degree of sorting, the degree of packing, the size and geometry of the voids, the hydraulic gradient, the presence of entrapped air, and temperature (Nelson and Baver 1940; Christiansen, 1944; Pillsbury and Appleman, 1950; Marshall, 1958; Jumikis, 1962; and Webb, 1969).

It is generally assumed that the rate of flow through a column of spheres is directly proportional to the square of the diameter of the spheres. This is because when the diameter of the spheres is doubled, the throat-plane area (the area of the spaces between the spheres) increases fourfold. The assumption is true in so far as the rate of flow is dependent on the size of the channelway (the passage along which a liquid may flow), (Fraser, 1935, p.962).

The following section gives an account of Darcy's law, its derivation, laboratory and field methods for measuring the permeability coefficient of soils.

Darcy's Law

Darcy's law (1856) can be regarded as the fundamental law for flow of water through soils. It is used not only in drainage problems but also in problems of unsaturated flow. The law is as follows:

$$\frac{Q}{t} = \frac{k \times A \times H}{L} \qquad (1)$$

where

k = the coefficient of permeability

Q = the quantity of water flowing through the soil in time t

L = the height of the soil sample

A = the area of the cross-section through which the water flows

H = the constant head of water (hydraulic head) operating over the height of the soil.

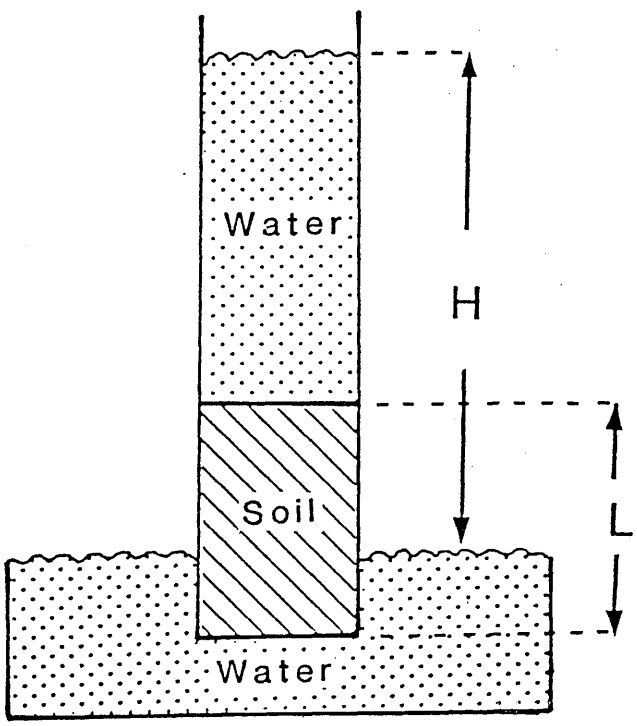
Those variables are shown in fig(1).

It is interesting to note the similarity between Darcy's law and other laws which govern physical processes. For example, Ohm's law, which relates to the flow of electricity through a conducting medium, is very similar to Darcy's law in that the flow of electricity is proportional to the voltage gradient and to the specific conductivity of the material (see app.1.1). In a similar way the flow of heat through a conducting solid is also proportional to the temperature gradient and to the property of the material known as thermal conductivity (Luthin, 1966). Another law related to Darcy's law is Fick's law (Mitchell, 1976), which describes the flow of ions.

Derivation of Darcy's law

Consider a cylinder of cross-sectional area A, length of sample L and hydraulic head H. A quantity of water Q passing through the sample in time t is collected in a measuring cylinder. The quantity of water Q is directly proportional to the time t, the cross-section area A and

Figure 1. Permeability measured by Darcy's law where H is the hydraulic head and L isthe height of the soil sample.


Water
H
Soil
L
Water

the hydraulic head H, and inversely proportional to the height of the sediment L. These statements can be written thus:

$$Q \propto t,\ Q \propto A,\ Q \propto \frac{1}{L},\ Q \propto H$$

Hence

$$Q \propto t \times A \times \frac{1}{L} \times H.$$

It therefore follows that:

$$Q = \frac{k \times t \times A \times H}{L}$$

where k is a constant of proportionality called the permeability coefficient. k can be found by rearranging equation (1) thus:

$$k = \frac{Q \times L}{t \times A \times H} \qquad (2)$$

Dimensional analysis of the units of permeability coefficient

Dimensional analysis of an equation in terms of mass (M), length (L) and time (T) enable one to calculate the units of the equation. This is done for the permeability coefficient, k, as follows:

The volume of water Q is measured in cubic units i.e. L^3

The time t is measured as T

The length of the sediment L is measured as L

The height of the hydraulic head H is measured as L

The area of the cylinder A is measured as L^2

Hence

$$k = \frac{L^3 \times L}{T \times L^2 \times L} = \frac{L}{T}$$

Thus the units of k are hence length/time which can be cm/sec, mm/hour, and so on.

Methods of measuring permeability

Most methods of measuring permeability depend on an application of Darcy's law. The following section gives an account of two of the more popular methods used in the laboratory. They are as follows:

a) Constant head permeameter

b) Variable head permeameter .

Field measurements can also be undertaken. These are:

1) When water table is present:

a) The auger hole methods

b) Well and pumping method

c) Multi layer method

2) When water table is absent:

c) Particle size method

d) Shallow water pumping test

e) The permeameter method

d) Pond-Infiltration test.

Laboratory measurements

The two most common laboratory methods for determining the coefficient of permeability of soils are as follows:

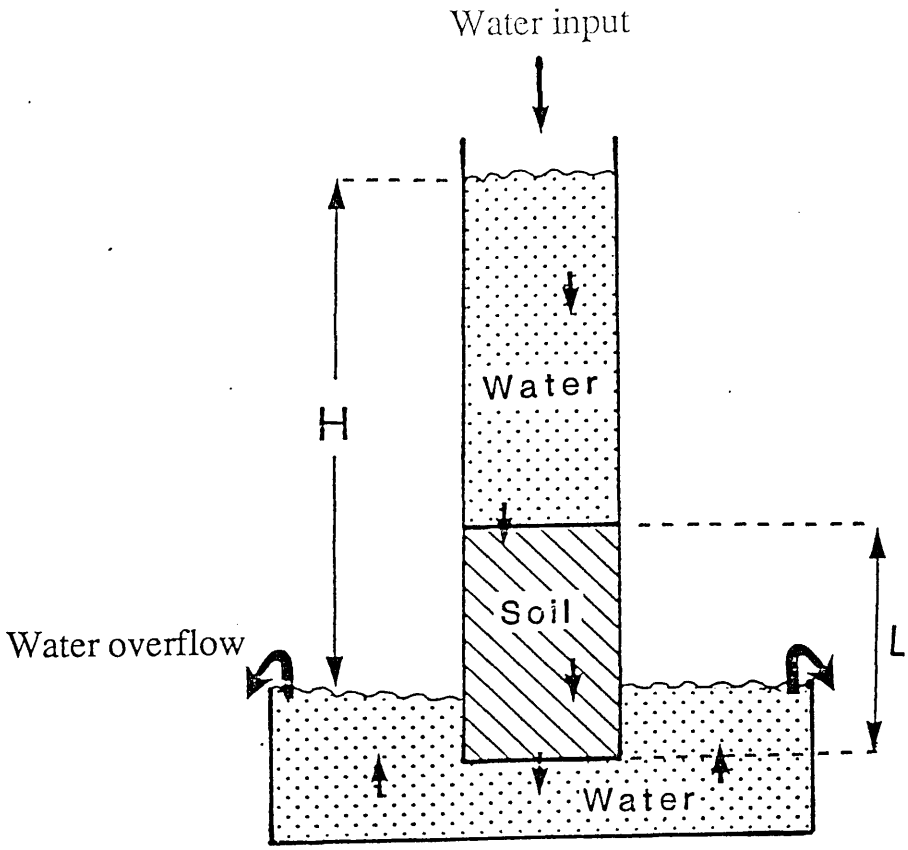
a) Constant head permeameter

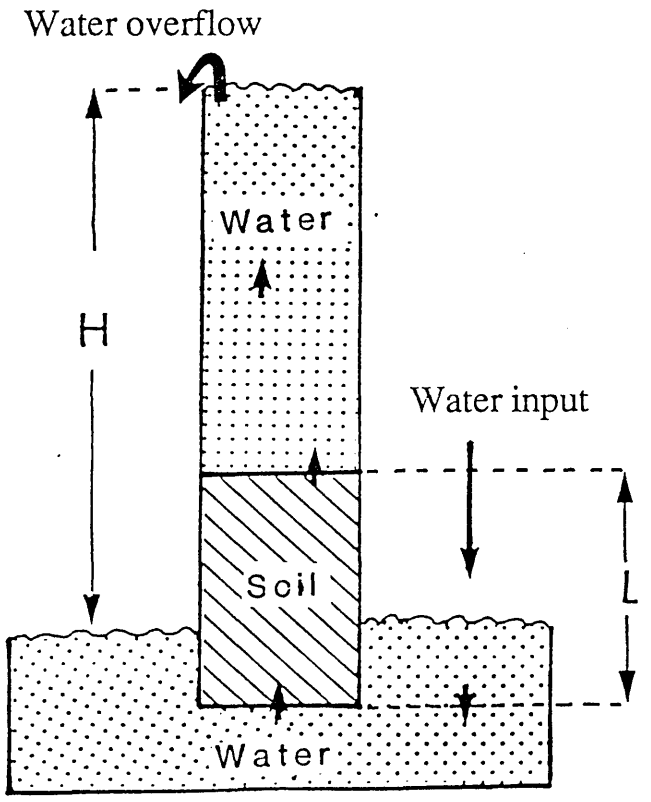
This test is suitable for more permeable granular soils or sediments. A given quantity of water Q is allowed to pass through the sample (fig,2a,2b). The level of the water is kept constant by addition of water, in other words there is a constant head of water pressure. Water passes through the sample in time t and is collected. From Darcy's law (Eq.1),

$$k = \frac{Q \times L}{t \times A \times H}$$

Figure 2a. Laboratory measurements using a constant-head permeameter for downward flow. The level of the water is kept constant by addition of water. H is the head of water and L is the length of the sample. Small arrows indicate direction of water flow. (Modified from Dunn, et al. 1980).

Figure 2b. Laboratory measurements using a constant-head permeameter for upward flow. The level of the water is kept constant by addition of water. H is the head of water and L is the length of the sample. Small arrows indicate direction of water flow. (Modified from Dunn, et al. 1980).


Water input
Water
H
Soil
Water overflow
L
Water


Water overflow
Water
H
Water input
Soil
L
Water

Most permeameters work by downward flow although an upward flow permeameter can also be used (fig.2a,2b). A sand filter is sometimes incorporated above and below the sample to avoid the soil or sediment being disturbed by the water flow (Israelsen and Hansen, 1962; Cedergren, 1977; Hansen, et al. 1980; Smith, 1981; Das, 1985).

b) Variable head permeameter

This test is more suitable for fine-grained soils or sediments. In the variable head permeameter water is also allowed to pass through the sample (fig.3). However the level of the water does not remain constant because no water is added to the cylinder containing the sediment. The time t for the water to fall from H_1 to H_2 is noted (fig,3). At any time (dt) the reduction in head is

$$H_1 - H_2 = - dH,$$

(the negative sign occurs because H decreases as time progresses). Hence the quantity of water flowing through the sample in time dt is:

$$Q = - A\ dH,$$

where

$$A = \pi r^2 ,$$

the area of the tube. From these measurements, the permeability coefficient can be calculated as:

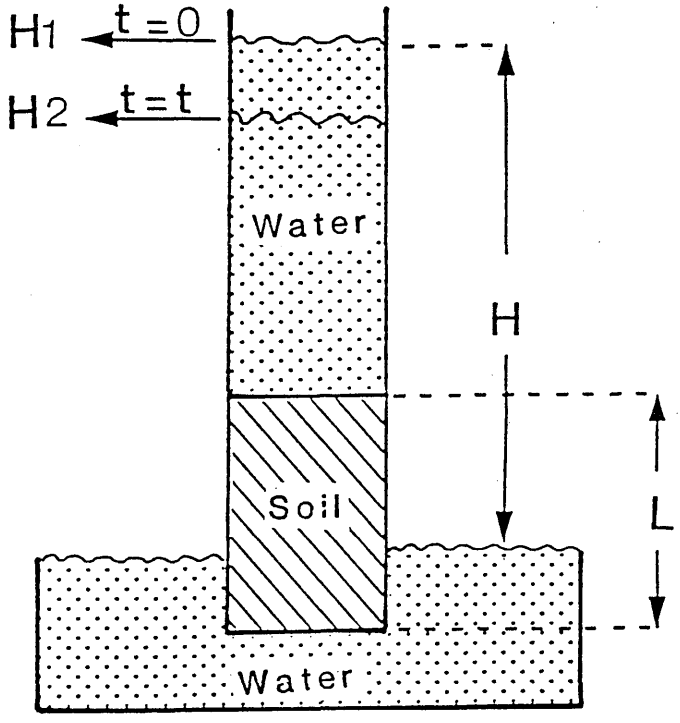
$$k = \frac{L \times \ln (H_1 / H_2)}{t} \qquad (3)$$

This equation is derived from Darcy's law (Eq.1) as follows:

$$\frac{Q}{t} = \frac{k \times A \times H}{L}$$

$$\frac{- A\ dH}{dt} = \frac{k \times A \times H}{L}$$

Figure 3. Laboratory measurements using a variable-head permeameter. The level of the water does not remain constant because no water is added to the cylinder containing the sediment. The time t for the water to fall from the head H_1 to H_2 is noted in the figure where H is the head of water and L is the length of the sediment. (Modified from Dunn et al. 1980).


H1
t = 0
H2
t = t
Water
H
Soil
L
Water

$$-A\,dH = \frac{k \times A \times H \times dt}{L}$$

$$dt = \frac{-(A \times L)\,dH}{A \times K \times H}$$

Integrating both sides of the equation between limits 0 to t and H_1 to H_2 respectively gives

$$\int_0^t dt = \int_{H_1}^{H_2} \frac{-L}{k \times H}\,dH$$

and since - L/K is a constant

$$\int_0^t dt = \frac{-L}{k}\int_{H_1}^{H_2} \frac{1}{H}\,dH$$

and so

$$\int_0^t t = \frac{-L}{k}\Big|_{H_1}^{H_2} \ln H$$

$$t = \frac{-L\,(\ln H_2 - \ln H_1)}{k}$$

$$t = \frac{L\,(\ln H_1 - \ln H_2)}{k}$$

$$t = \frac{L \times \ln (H_1 / H_2)}{k}$$

$$k = \frac{L \times \ln (H_1 / H_2)}{t}$$

as required (Cedergren, 1977; Smith, 1981; Das, 1985).

Field measurements

Field methods of measuring the permeability of soils and sediments are relevent to:

1) Land drainage (highways, airport and buildings)
2) Irrigation of deserts for land reclamation and agricultural use
3) Foundations of buildings
4) Buried nuclear waste
5) Seepage through and under earth dams
6) Agriculture and soil use in temperate climates
7) Oil platforms
8) Intertidal shores and supralittoral sand dunes
9) Well construction
10) Irrigation of sands and soil.

Soil permeability in the field can be measured when the water table is absent and when it is present.

For land drainage, irrigation of deserts and foundations of buildings we can measure the permeability when a water table is absent.

When a water table is present the permeability can be measured for soil used in the following areas:

1) Buried nuclear waste
2) Seepage through and under earth dams
3) Dams
4) Oil platforms
5) Intertidal shores
6) Wells
7) Irrigation of sands and soil.

The following paragraphs give the field measurements when the water table is present and when it is absent.

I) Water table present.

1) THE AUGER HOLE METHODS

A hole is dug in the soil or sediment using a hand held auger or a mechanical device. The bottom of the hole must be below the water table.

After allowing the water level in the hole to come into equilibrium with the water table, the water is pumped out of the hole, and measurements are then made of the rate of rise of water in the hole. These measurements are used to calculate the permeability coefficient of the sediment using Hooghoudt's equation. There are also three other methods using similar auger hole methods which I shall refer to, but I do not propose to describe them in detail.

(i) Hooghoudt(1936)

Hooghoudt (1936) mathematically analysed the auger hole method in a homogeneous soil based on his own experimental observations. I have had problems with Hooghoudt's (1936) paper because it is in Dutch, is long (89 pages), and is somewhat confusing. In addition, all the references that I have been able to find to it in the literature give the appearance of these authors having had similar problems.

The following account is taken partly from Hooghoudt's (1936) original paper and partly from other literature references to it which are as follows:

Luthin (1966).

Taylor and Ashcroft (1972).

Raudkivi and Callander (1976).

Dunn, et. al. (1980).

Hooghoudt (1936) made two assumptions in his analysis:

1) The water table around the auger hole is not lowered when water is pumped out of the hole. This condition is approximately satisfied for a short period after the initiation of pumping from the auger hole. If the auger hole is pumped repeatedly, however, this condition may not be met.

2) Water flows horizontally into the sides of the auger hole and vertically up through the bottom of the hole (figure 4). This is always true except where the bottom of the hole is resting in an impervious layer whereupon the mathematics are slightly modified.

Hooghoudt's equation has an empirical factor (s) defined by experiment (Hooghoudt, 1936; Luthin, 1966) (see below). Luthin (1966) states that S should depend on r, the radius of the auger hole, H the distance from the bottom of the hole to the water table, and on s the height between the bottom of the hole and the impermeable layer. However Hooghoudt (1936) gave S as:

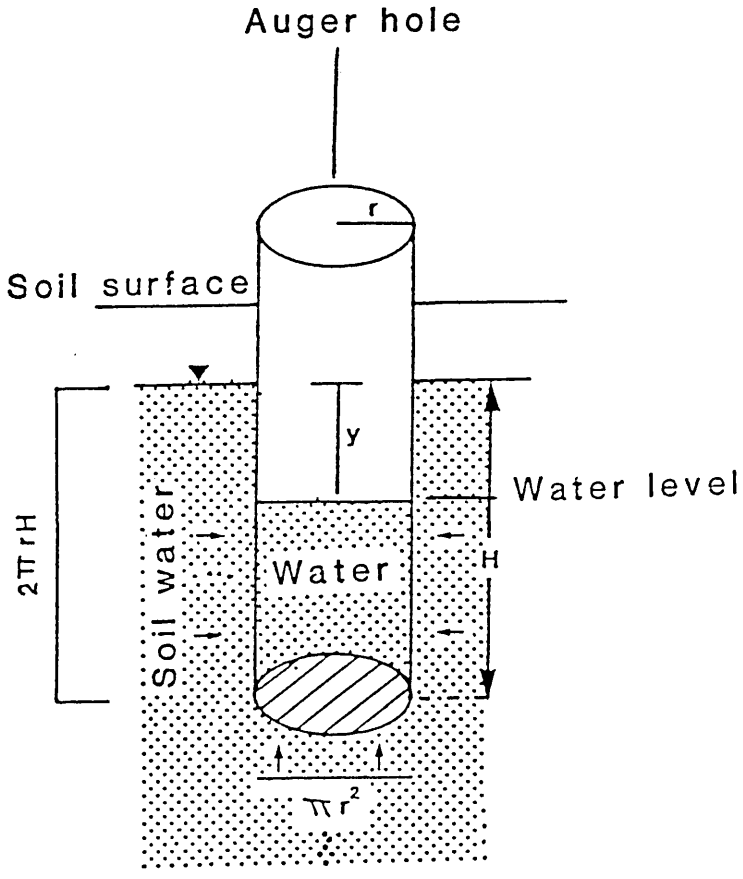
$$S = \frac{rH}{0.19}$$

where S has the dimension of a length, but in which s and the height of the water (H-y) in the hole do not occur. According to Luthin (1966) there appears to be no obvious reason for the absence of s and (H-y) from S.

Hooghoudt's (1936) determination of S

Hooghoudt (1936) determined S with the aid of a controlled experiment in a sand tank 10 metres long, 2 metres wide and 2 metres deep. This tank was filled with river sand. A series of tubes showing the level of the ground water were placed in a row in the middle

Figure 4. Water flows horizontally into the sides of the auger hole and vertically up through the bottom of the hole. $2\pi rH$ is the surface area of the walls and πr^2 is the surface area of the base of the hole. y equals the vertical height between the water in the sediment and the water level in the auger hole. H is the hydraulic head and r is the radius of the hole.


Auger hole
r
Soil surface
y
Water level
2π rH
Soil water
Water
H
πr²

between the long sides of the tank. These tubes were constructed from perforated copper pipe and had a diameter of 4 centimeters. In the first experiment one of the tubes in the middle of the reservoir had sand removed from it and was used as an auger hole. The second experiment was carried out using an auger hole of 11 cm diameter, which had been bored into the sand in the centre of the tank at well over 60cm from one of the long sides. This auger hole was protected by a perforated tube of the same diameter, to avoid infill from the sides.

Hooghoudt then measured the rate at which water flowed into the auger holes in the two experiments, and from these and other observations was able to derive S. The experimental conditions under which Hooghoudt determined S only approximated field conditions because of the finite size of the sand tank used, and Hooghoudt suggests that the coefficient 0.19 is only accurate to within about 27% of the true value. He considers the accuracy adequate for the determination of hydraulic conductivity (permeability coefficient) which may vary in the field from 0.001 to more than 10 m/day. The numerical coefficient 0.19 has the dimension of length and is valid only for metres; it is used in all recent accounts.

Derivation of Hooghoudt's (1936) auger hole equation

There are two routes by which the Hooghoudt's equation can be developed. The first is by considering the rate of decrease of y in relation to time, where y equals the vertical height between the water table in the soil or sediment and the water level in the auger hole (Luthin 1966). The second is by considering the rate of increase of V, the volume of water in the auger hole in relation to time (Raudkivi and Callander 1976). The two methods converge to the

same equation.

Both methods rely on the two assumptions made by Hooghoudt (1936) (see above).

Method 1 (Hooghoudt 1936; Luthin 1966)

The rate at which the auger hole fills with water is proportional to the surface area of the walls and base of the hole across which the water is flowing:

$$-\frac{dy}{dt} \propto (2\pi rH + \pi r^2)$$

and to the difference between the water table and the water level in the hole

$$-\frac{dy}{dt} \propto y$$

It is also inversely proportional to the cross-sectional area of the hole,

$$-\frac{dy}{dt} \propto \frac{1}{\pi r^2}$$

Hooghoudt (1936) also states that $-\frac{dy}{dt}$ is inversely proportional to S. (where $S = \frac{0.19}{rH}$).

$$-\frac{dy}{dt} \propto \frac{1}{S}$$

Hence

$$\frac{dy}{dt} \propto \frac{2\pi rH + \pi r^2}{\pi r^2} \cdot \frac{y}{S}$$

and

$$-\frac{dy}{dt} = K \frac{2\pi rH + \pi r^2}{\pi r^2} \cdot \frac{y}{S}$$

where K is the permeability coefficient.

This equation can now be simplified to:

$$-\frac{dy}{dt} = K \left(\frac{2\pi rH}{\pi r^2} \cdot \frac{y}{S} + \frac{\pi r^2}{\pi r^2} \cdot \frac{y}{S}\right)$$

$$-\frac{dy}{dt} = K\left(\frac{2H}{r}\,\frac{y}{S} + \frac{y}{S}\right)$$

$$dy = -K\left(\frac{2H + r}{rS}\right) y \cdot dt \qquad (4)$$

If we integrate equation 4 with respect to y and t between the limits $y = y_0$ to $y = y$ and $t = 0$ to $t = t$, We obtain

$$\int_{y_0}^{y} \frac{1}{y}\,dy = -K\,\frac{2H + r}{rS}\int_{0}^{t} dt$$

$$\ln\frac{y_0}{y} = K\,\frac{2H + r}{rS}\,(-t)$$

$$\ln\frac{y_0}{y} = K\,\frac{2H + r}{rS}\,t$$

$$K = \frac{rS}{(2H + r)t}\ln\frac{y_0}{y} \qquad (5)$$

which is Hooghoudt's (1936) equation. Infact, Hooghoudt (1936, pp.16-17) uses this method to derive his equation.

Method 2 (Raudkivi and Callander, 1976)

Again, the rate at which the auger hole fills with water is proportional to the surface area of the walls and base of the hole across which the water is flowing ($2\pi rH + \pi r^2$), and to the difference between the water table and the water level in the hole (y). But rather than considering $\frac{dy}{dt}$, Raudkivi and Callander (1976) consider $\frac{dv}{dt}$ as follows.

If dv is the volume of water flowing into the hole in time dt:

$$\frac{dv}{dt} \propto (2\pi rH + \pi r^2) \quad y$$

as in method 1.

Hence

$$\frac{dv}{dt} = K\,(2\pi rH + \pi r^2)\quad y \qquad (6)$$

where K again is the permeability coefficient.

In addition, the volume of water entering the hole dv, in time dt is the product of the cross sectional area of the hole and the increase in water level dy in time dt. Therefore,

$$dv = \pi r^2\ (-dy)$$

The negative sign occurs because as the water level increases with time, the distance between the water level and water table decreases (y). Substituting $dv = \pi r^2\ (-dy)$ into equation 6 gives

$$\pi r^2\ \left(-\frac{dy}{dt}\right) = K\,(2\pi rH + \pi r^2)\quad y$$

As in method 1,

$$-\frac{dy}{dt} = K\,\frac{2\pi rH + \pi r^2}{\pi r^2}\,\frac{y}{S}$$

where S is introduced at this stage

$$-\frac{dy}{dt} = K\,\frac{2H + r}{rS}\,y \qquad (7)$$

Equation 7 is the same as equation 1 and it will integrate to give equation 5

$$K = \frac{rS}{(2H + r)t}\,\ln\frac{y_0}{y}$$

(ii) The Two Auger Hole Method

Childs (1952) and Childs, et al. (1953) have proposed a method for nonlayered soil using two auger holes. The two holes are of equal diameter and penetrate to the same depth below the water table.

They consider two methods, one reaches an impermeable layer and the other does not.

Method one

Water is pumped at a steady rate out of one hole and carried by a hose into the other thus creating a small hydraulic head difference between the levels of water in the two holes (figure, 5).

Childs (1952) and Childs, et al. (1953) derive the permeability coefficient K as

$$K = \frac{Q}{\pi L dH} \cosh^{-1} \frac{b}{2r}$$

where

Q = the pumping rate

dH = the hydraulic head difference between the two holes

L = the depth of each hole below the water table

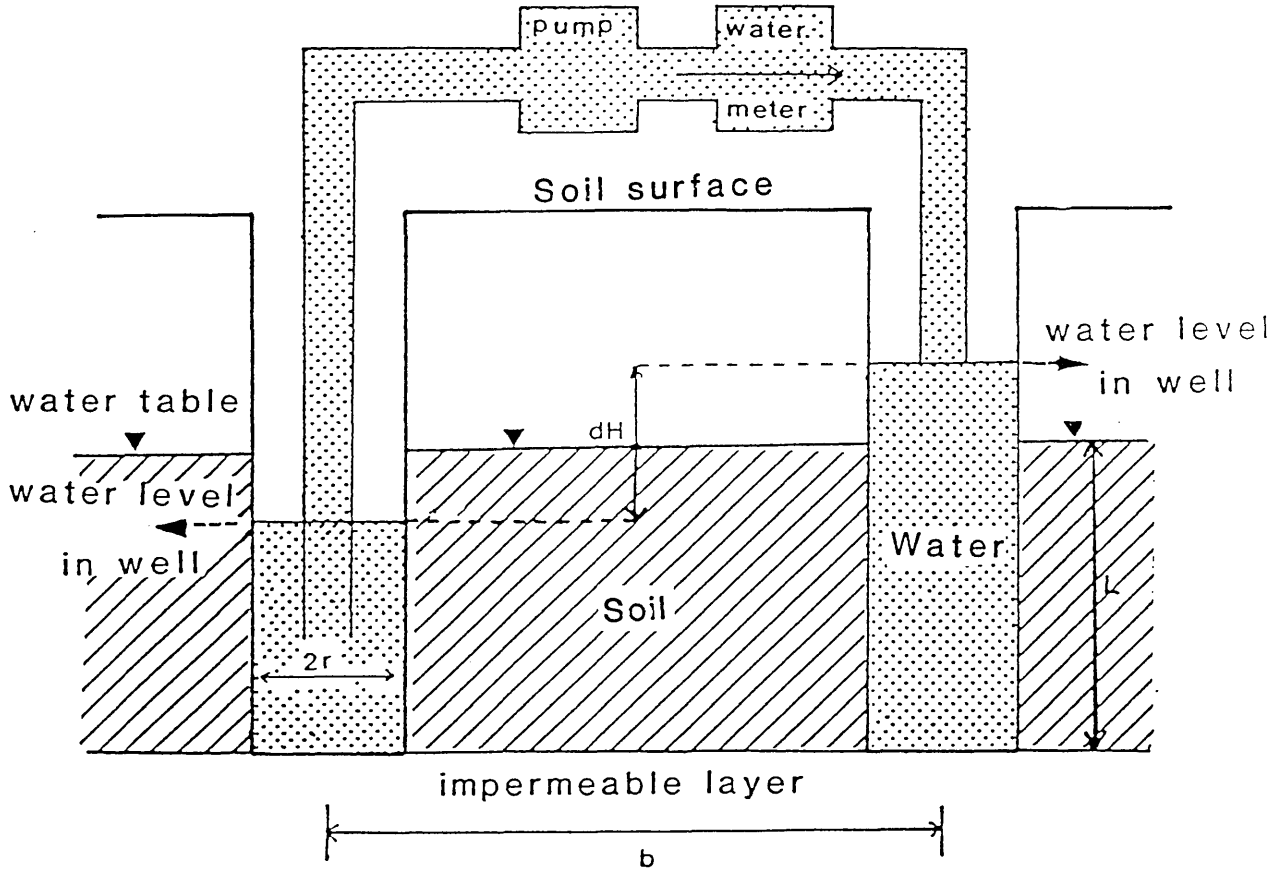
r = the radius of each hole

b = the distance between their vertical axes.

Method two

If the auger hole does not reach an impermeable layer, an end correction must be applied to compensate for the flow entering the end of the auger hole. The end effect may be regarded as a flow which extends the length of the auger hole and depends on the depth to the impermeable layer as well as on the dimensions of the hole. An addition of some 20 cm to the measured depth is suggested by Childs, et al.(1953) as an appropriate end correction for holes of the radius they used. In addition the effective flow region between the two holes is enlarged by the flow which occurs in the capillary

Figure 5. The two auger holes method (Luthin 1966). The two holes are of equal diameter and penetrate to the same depth below the water table. dH is the hydraulic head difference between the two holes, L is the depth of each hole below the water table, r is the radius of each hole and b is the distance between their vertical axes.


pump
water
meter
Soil surface
water level
in well
water table
dH
water level
in well
Water
Soil
2r
L
impermeable layer
b

fringe. Once again it is possible to compensate for this flow by extending the effective length of the auger hole. Adding 5 cm to L will usually be adequate. Alternatively, it is possible to make an estimate of the capillary fringe in the field and to take half of the thickness of the capillary fringe as the fringe correction. The capillary fringe is a region of uniform moisture content above the water table, with a conductivity or permeability that is essentially the same as the hydraulic conductivity or permeability of the saturated soil (Childs 1945a,b; Childs et al.1953; Luthin 1966). According to Tolman (1937), the capillary fringe can also be regarded as the "zone immediately above the water table in which water is held above the water table by capillarity".

(iii) The pipe cavity method

The method, which was proposed by Kirkham (1949), is conducted as follows. Tubes or pipes can be pushed vertically into the soil below the water table either with or without a cavity at the end of the tube. The soil is augered out of the tube and the water table allowed to establish itself. Water is pumped out of the tube in order to measure the soil permeability. The rate of rise of water in the hole can be used to calculate the soil permeability using Kirkham's (1946) equation (figure 6).

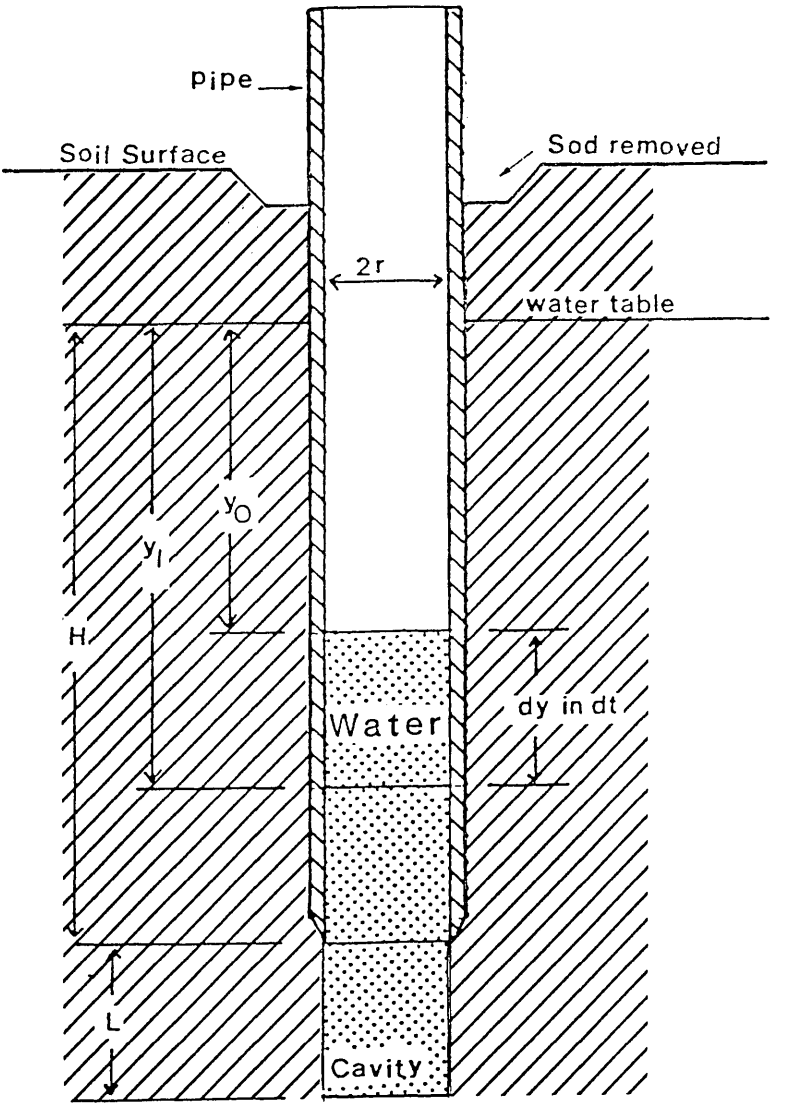
$$K = \frac{\pi r^2 \ln \frac{y_0}{y_1}}{S(t_2 - t_1)}$$

where

K = permeability coefficient

y_0 = distance from water table to water level in tube or pipe at time t_2

Figure 6. The pipe cavity method (Luthin 1966) consists of pipes pushed vertically into the soil below the water table. y_0 and y_1 are the distances from the water table to the water levels in the pipe at time t, r is the radius of the pipe, H is the hydraulic head and L is the cavity length.


Pipe
Soil Surface
Sod removed
2r
water table
y_o
y_1
H
Water
dy in dt
L
Cavity

y_1 = distance from water table to water level in pipe at time t_1

r = radius of pipe

$t_2 - t_1$ = time for water level to change from y_0 to y_1

S = a coefficient determined by using an electric analogue (see app.1.1).

The S-factor is computed from the formula (Luthin and Kirkham, 1949):

$$S = \frac{r_c}{r_m} \cdot \frac{1}{k'R}$$

where

r_c = radius of the soil cavity

r_m = radius of cylindrical electrode

k' = specific conductivity of the tank electrolyte

R = electrical resistance between the cylindrical electrode and the copper tank bottom.

The field procedures used in the pipe cavity method consist of pushing a pipe into an auger hole slightly smaller in diameter than the pipe, using a special technique designed to eliminate compaction.

When the water table has to establish itself in the pipe, a hose connected to a pump is inserted into the pipe and the water is pumped out. The purpose of the pumping is to remove puddled soil from the walls of the cavity. The inseeping water flushes out the soil pores. After puddling effects have been reduced, the soil water is allowed to rise in the pipe, and the rate of rise is determined with the aid of stop watches and an electrical probe.

A simplified form of Kirkham's equation can be used for the actual calculation if the measurements are made while the water level in the pipe is less than half the distance to the water table.

Luthin (1966, p.142) gives an example of the simplified equation which he says is only valid for a given set of conditions. These conditions are a 5.08cm pipe having a cavity 10.16cm long and 4.83cm diameter, and S being 43.18cm. This equation is:

$$K = 655 \frac{dy}{dt} \frac{1}{\text{average head}}$$

(iv) Ernst's Formula

Ernst (1950) studied the auger hole problem and with the aid of numerical analysis, he developed some empirical equations which can be used to solve the auger hole problem. The following formula was obtained in the case of homogeneous soil with an impermeable layer at a great depth below the bottom of the auger hole.

$$k = \frac{4000}{\left(20 + \frac{d}{a}\right)\left(2 - \frac{y}{d}\right)} \frac{a}{y} \frac{\Delta y}{\Delta t} \qquad (4)$$

This formula k is expressed in meters per 24 hrs. All other quantities are in centimeters or in seconds.

k = hydraulic conductivity

Δy = rise of water surface in auger hole during the time interval Δt

d = depth of water in hole before pumping

y = distance from static water table to elevation of water in the hole

a = radius of auger hole.

According to Ernst (1950) the value of k is approached with an accuracy of ± 20% if the following conditions are met:

$$3 < a < 7 \text{ cm}$$

$$20 < d < 200 \text{ cm}$$

$$0.2 < \frac{y}{d} < 1$$

$$d < s$$

In the above equation, s represents the difference in height between the bottom of the hole and the impermeable layer. Ernst points out that the measurement should not be continued for too long a period of time since the funnel shaped curve of the water table around the hole may become too great for the results to hold. Measurements should end before 25% of the volume of water removed from the hole has flowed back into the hole. In other words, the measurements should be completed before $y_n > (3/4)\ y_1$ where y_n is the height of water in the hole at the time of the nth measurements.

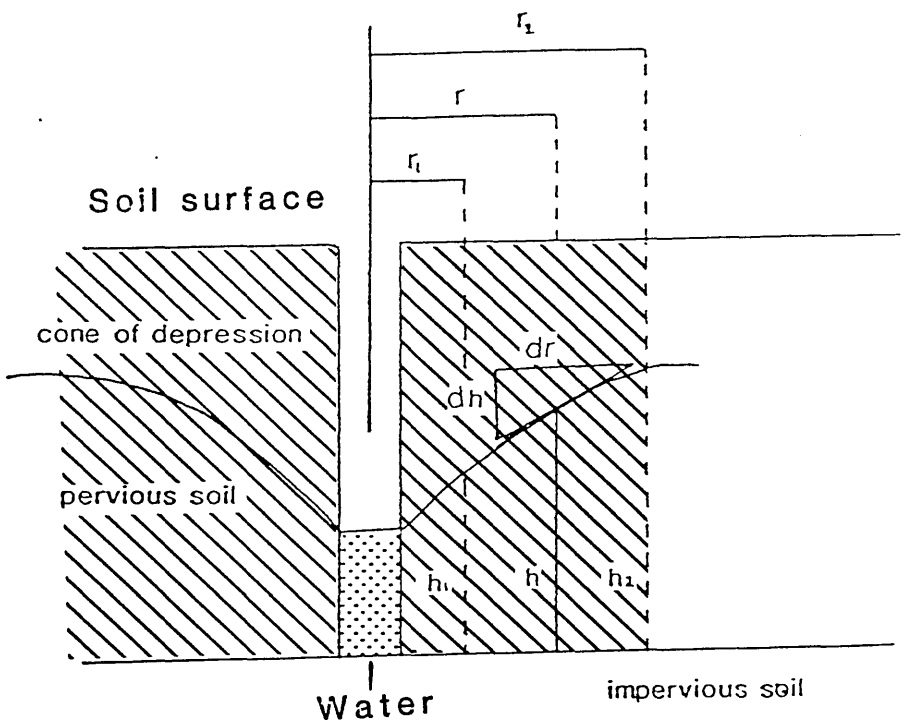
Usually Δy and Δt are measured several times to increase the accuracy of the results and to reduce the influence of irregularities. Before making a new run of $\Delta y/\Delta t$ against y, the static water table should be allowed to establish itself, otherwise the funnel-shaped depression around the auger hole will result in apparently smaller values of k on succeeding runs following pump outs (Ernst, 1950; Luthin, 1957; Dunn, et al. 1980).

2) Well and pumping method-wells equation

When water is pumped from a well point the water is lowered adjacent to the point, giving a cone of depression. This cone of depression will form even in relatively impervious soils after sufficient time has elapsed.

Consider the horizontal flow of water through a thin element of the soil of distance r from the well point where the head of water above an impervious layer is h (figure 7).

Figure 7. Horizontal flow of water through a thin element of the soil at different distances r_1, r and r_2 from the well point where the heads of water lie above an impervious layer are h_1, h and h_2. dH/dr is the rate of change of head per unit length (Dunn, et al. 1980).


r₂
r
r₁
Soil surface
cone of depression
dr
dh
pervious soil
h₁
h
h₂
Water
impervious soil

The rate of flow, $\frac{Q}{t}$, is the rate at which water is being pumped and may readily be determined.

The surface area of the soil element at distance r from the well point is equal to the surface area of a cylinder radius r height h, or $A = 2 \times \pi \times r \times h$.

The hydraulic gradient at that point is the rate of change of head per unit length or, $i = \frac{dh}{dr}$. From Darcy's law:

$$\frac{Q}{t} = K \times A \times i$$

$$\frac{Q}{t} = K \times (2\pi rh) \times \left(\frac{dh}{dr}\right)$$

or

$$\frac{dr}{r} = K \times \left(2\pi \Big/ \left(\frac{Q}{t}\right)\right) \times h \, dh$$

Integrating between limits r_1 to r_2 and h_1 to h_2 :

$$\int_{r_1}^{r_2} \frac{dr}{r} = K \times \frac{2\pi}{\frac{Q}{t}} \int_{h_1}^{h_2} h \, dh$$

$$\ln(r) = K \times \frac{2\pi}{\frac{Q}{t}} \times \frac{h^2}{2}$$

$$\ln(r_2) - \ln(r_1) = K \times \frac{2\pi}{Q/t} \times \frac{h_2^2 - h_1^2}{2}$$

$$\ln\left(\frac{r_2}{r_1}\right) = K \times \frac{2\pi}{Q/t} \times \frac{h_2^2 - h_1^2}{2}$$

$$K = \ln\left(\frac{r_2}{r_1}\right) \times \frac{1}{\frac{2\pi}{Q/t}} \times \frac{2}{h_2^2 - h_1^2}$$

Hence the coefficient of permeability is:

$$K = \ln\left(\frac{r_2}{r_1}\right) \times \frac{\frac{Q}{t}}{\pi(h_2^2 - h_1^2)}$$

Hence, by measuring the height of the ground water at two points distances r_1 and r_2 from the well point, the coefficient of permeability may be determined.

Considerable practical experience is required before a reliable result by this method can be obtained, since the soil is unlikely to be homogeneous, and include impervious layers and not likely to be horizontal. Pumping should be allowed to continue until conditions are settled before any measurements are taken, and the observation wells must not be too close to the well point or else soil will be disturbed and the drop in head will be artificially rapid.

The derivation of the equation for determining the coefficient of permeability for a permeable stratum thickness d and overlain by a relatively impervious stratum (figure 8) is as follows. From Darcy's law:

$$\frac{Q}{t} = K \times A \times \frac{H}{L}$$

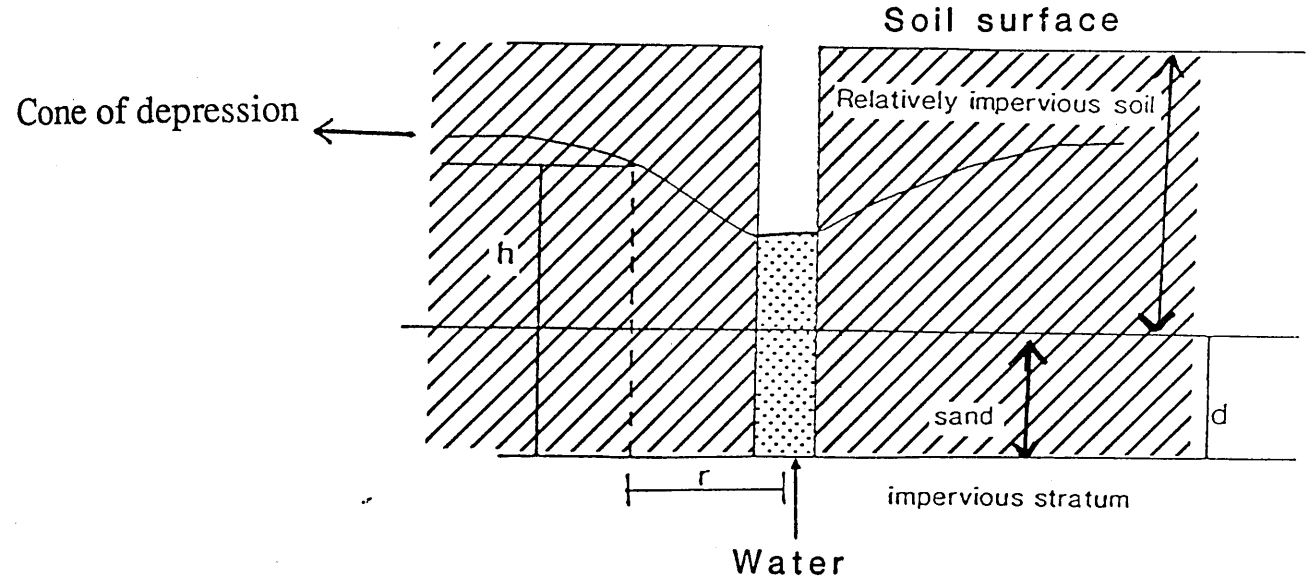
and

$$\frac{Q}{t} = K \times A \times \frac{dh}{dr}$$

Integrating between limits r_1 to r_2 and h_1 to h_2 :

$$\int_{r_1}^{r_2} \frac{dr}{r} = K \times (2\pi d) \times \frac{1}{Q/t} \int_{h_1}^{h_2} dh$$

Figure 8. Thin element of soil (in this case a permeable sand) of distance r from the well point overlain by a relatively impervious stratum of soil. The thickness of the permeable stratum (sand) is d (Smith 1981).


Soil surface
Cone of depression
Relatively impervious soil
h
sand
d
r
impervious stratum
Water

$$\int_{r_1}^{r_2} \ln(r) = K \times (2\pi d) \times \frac{1}{Q/t} \times \int_{h_1}^{h_2} h$$

$$\ln\left(\frac{r_1}{r_2}\right) = K \times (2\pi d) \times \frac{1}{Q/t} \times (h_2 - h_1)$$

Then the coefficient of permeability is

$$K = \frac{\ln\left(\frac{r_2}{r_1}\right) \times \frac{Q}{t}}{2\pi d \times (h_2 - h_1)}$$

(Smith, 1981).

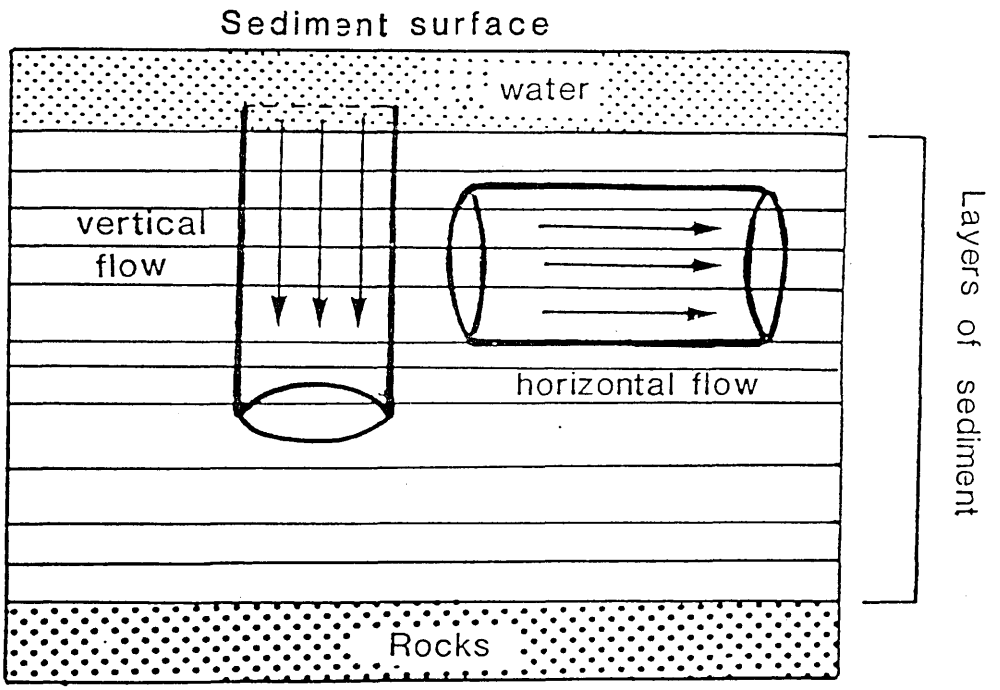
3) Multi layer permeability

Frequently the soil mass through which seepage takes place consists of several strata with different coefficients of permeability. Even when the soil is of fairly uniform composition the permeability tends to decrease with depth because of the increasing density caused by the progressive weight of the strata above. In many natural soil deposits the permeability in the horizontal direction is several times that in the vertical direction.

When the permeability varies with depth, a convenient approximation is to find an equivalent coefficient of permeability for horizontal flow and another equivalent value for vertical flow (figure 9). These coefficients, though differing from one another, are assumed to be constant throughout the depth of the soil mass under consideration (Capper and Cassie, 1976, page 51).

Smith, (1981) and Capper and Cassie, (1976) each give two equations to determine the permeability of different layers of sediment, one for horizontal flow and the other for vertical flow. In the following paragraph I am going to state the equation for horizontal flow firstly, and then for vertical flow.

Figure 9. Diagram of water flow through layers of sediment in vertical and horizontal directions.


Sediment surface
water
vertical
flow
horizontal flow
Rocks
Layers of sediment

Horizontal flow

Smith's (1981) equation for horizontal flow is:

$$k_H = \frac{k_1 H_1 + k_2 H_2 + \ldots + k_n H_n}{H_1 + H_2 + \ldots + H_n}$$

where

k_H is the average permeability in horizontal flow

$k_1, k_2, \ldots, k_n$ are permeabilities in a series of strata

$H_1, H_2, \ldots, H_n$ are the thickness of the layers.

Capper and Cassie's (1976) equation for horizontal flow is:

$$K_x = \frac{1}{Z}\int_0^Z K\, dz = \text{mean ordinate of the curve showing K against z.}$$

where

K_x is the equivalent coefficient of permeability for horizontal flow

K is the coefficient of permeability in the horizontal direction at any depth z.

I have been able to equate these two equations as follows:

Let K be the coefficient of permeability in the horizontal direction at any depth z. The flow in a horizontal direction through an element of unit width and thickness dz (figure 10) is given by Capper and Cassie, (1976) as

$$dq = Ki\, dz \qquad (8)$$

where i is the hydraulic gradient. The total flow through a section of unit width and depth Z is

$$q = K_x i\, Z \qquad (9)$$

Integrating equation 8, gives

$$\int dq = \int_0^Z K\, i\, dz$$

$$q = i \int_0^Z K\, dz. \qquad (10)$$

Equation 9 is equivalent to equation 10, hence

$$K_x \; i \; Z = i \int_0^Z K \, dz$$

Therefore,

$$K_x = \frac{i}{i \; Z} \int_0^Z K \, dz$$

$$K_x = \frac{1}{Z} \int_0^Z K \, dz \qquad (11)$$

where K_x is the average permeability for horizontal flow. This is the equation given by Capper & Cassie, (1976, page 52) for the horizontal plane. Figure 11, shows a plot of K against Z and the area under the curve. Where

$$\int_0^Z K \, dz = k_1 \; z_1 \; + k_2 \; z_2 \; + \ldots + k_n \; z_n \qquad (12)$$

This plot shows that permeability decreases with increasing in depth. Therefore,

$$K_x = \frac{1}{Z} \int_0^Z K \, dz = \frac{k_1 \; z_1 + k_2 \; z_2 + \ldots + k_n \; z_n}{z_1 + z_2 + \ldots + z_n}$$

where $Z = z_1 + z_2 + \ldots + z_n$

The left hand side of this equation is Capper & Cassie's (1976, page 52) equation, and the right hand side is Smith's (1981, page 45) equation.

Vertical flow

Smith's, (1981) equation for vertical flow is:

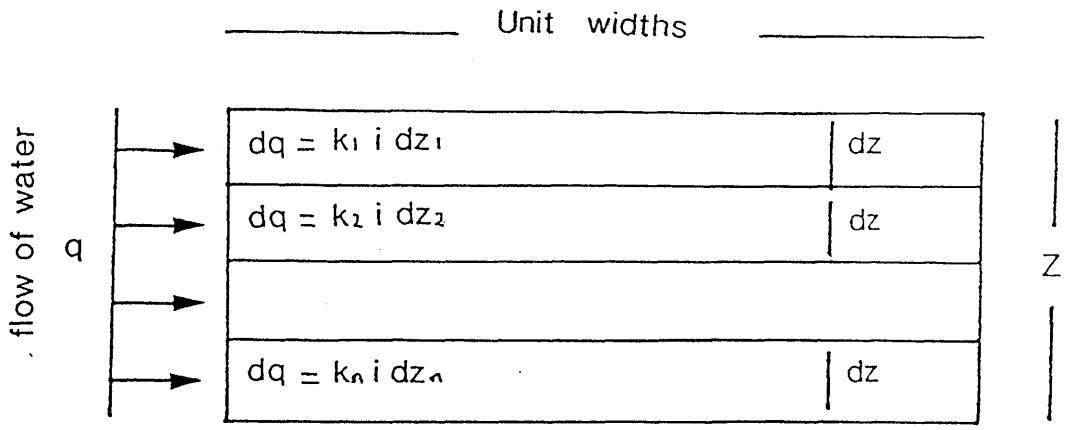
$$Kv = \frac{H_1 + H_2 + \ldots + H_n}{\frac{H_1}{K_1} + \frac{H_2}{K_2} + \ldots + \frac{H_n}{K_n}}$$

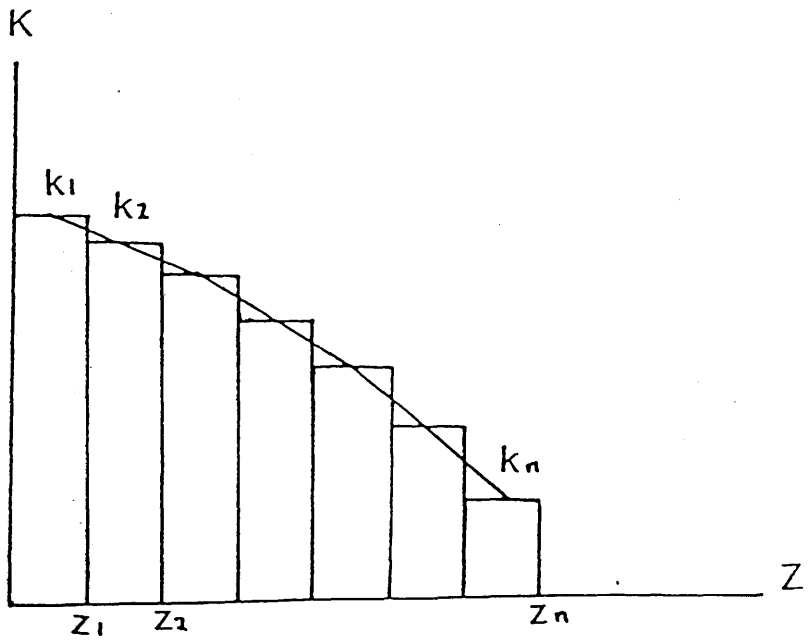
Figure 10. Flow in a horizontal direction through an element of soil of unit width and thickness z. The flow (dq) in a horizontal direction through an element of unit width and thickness dz is

$dq = k_1 \, i \, dz_1$, $dq = k_2 \, i \, dz_2$, ... , $dq = k_n \, i \, dz_n$

k_1, k_2, ... , k_n are the permeability coefficients in a series of soil strata having thickness of layers dz_1, dz_2, ..., dz_n, and i is the hydraulic gradient. (Original diagram).

Figure 11. Plot of K against Z and the area under the curve. Permeability decreases with increasing soil depth. k_1, k_2,..., k_n are the permeability coefficients in a series of soil strata having thickness of layers z_1, z_2,..., z_n. (Modified from Capper and Cassie's (1976) equation).


Unit widths
flow of water
q
dq = k1 i dz1
dz
dq = k2 i dz2
dz
dq = kn i dzn
dz
Z


K
k1
k2
kn
Z
z1
z2
zn

Capper & Cassie's, (1976) equation for vertical flow is:

$$\frac{1}{K_z} = \frac{1}{Z} \times \text{area of curve of } \frac{1}{K} \text{ plotted against z.}$$

I have adopted the same method as for horizontal flow, as follows.

Let H be the head lost over a depth Z, let dh be the head lost in passing through an element of thickness dz and let the permeability of this element in the vertical direction be K (figure 12). Then

$$v = K \left(\frac{dh}{dz}\right).$$

where v is the flow of water. The total flow through a section of unit width and depth Z is

$$v = K_z \left(\frac{H}{Z}\right)$$

To find the equivalent permeability K_z put

$$v = K_z \left(\frac{H}{Z}\right) = K \left(\frac{dh}{dz}\right)$$

Therefore,

$$\frac{dh}{K_z} = \frac{H\,dz}{ZK} \qquad (13)$$

Integrating equation 13 between limits 0 to H and 0 to z

$$\int_0^H \frac{dh}{K_z} = \int_0^z \frac{H\,dz}{ZK}$$

gives

$$\frac{H}{K_z} = \frac{H}{Z} \int_0^z \frac{1}{K}\,dz$$

Figure 13 shows a plot of $\frac{1}{K}$ against Z and the area under the curve, where

$$\int_0^z \frac{1}{K}\,dz = \frac{z_1}{k_1} + \frac{z_2}{k_2} + \ldots + \frac{z_n}{k_n}$$

Therefore,

$$K_z = \frac{HZ}{H\int_0^z \left(\frac{1}{K}\right) dz} = \frac{z_1 + z_2 + \ldots + z_n}{\frac{z_1}{k_1} + \frac{z_2}{k_2} + \ldots + \frac{z_n}{k_n}}$$

where

$$Z = z_1 + z_2 + \ldots + z_n$$

The left hand side of this equation is Capper & Cassie's (1976, p.52) equation, and the right hand side is Smith's (1981, p.46) equation.

As in Capper and Cassie's, (1976) book:

$$\frac{1}{Kz} = \frac{1}{Z} \times \text{area of curve of } \frac{1}{k} \text{ ploted against z.}$$

II) Water Table Absent

1) Particle size method

In the absence of a measure of permeability, the coefficient for clean granular soil can be estimated from the following equation (Dunn et. al. 1980):

$$K = \frac{2g}{C_s}\,\frac{\rho}{\mu}\,D^2\,\frac{e^3}{1+e}$$

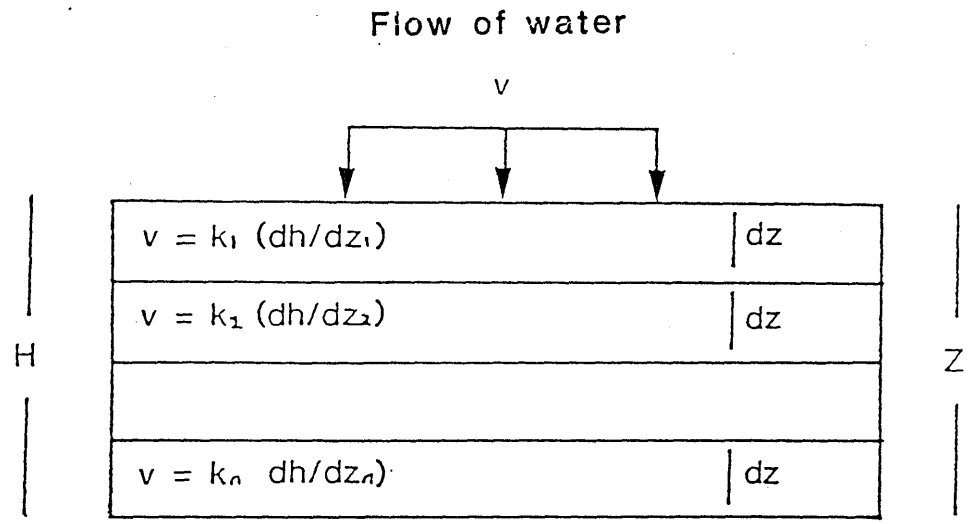
where

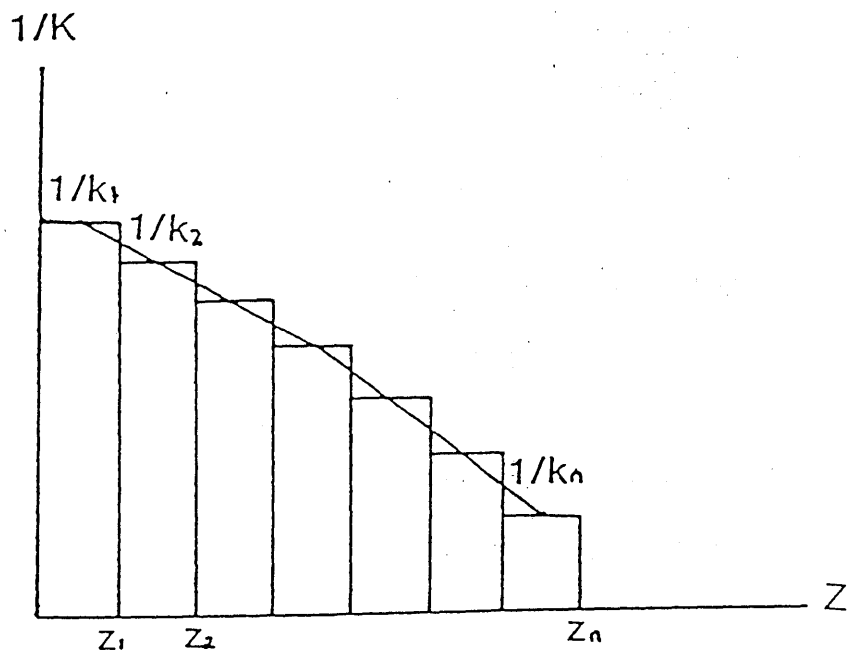
g = acceleration of gravity (9.81 m/sec^2)

$\frac{\mu}{\rho}$ = Kinematic viscosity of water (varies approximately linearly from 1.3 mm^2/sec at 10°C to 1.01 mm^2/sec at 20°C) (see app.1.2)

Figure 12. Flow (v) in a vertical direction through an element of soil of thickness z. H is the head lost over a depth Z, dh is the head lost in passing through an element of thickness dz and the flow of water v equals k_1 (H/z_1), k_2 (H/z_2), ..., k_n (H/z_n). k_1, k_2,..., k_n are the permeability coefficients in a series of soil strata having thickness of layers dz_1, dz_2,..., dz_n. (Original diagram).

Figure 13. Plot of 1/K against Z and the area under the curve. Permeability decreases with increasing soil depth. k_1, k_2,..., k_n are the permeability coefficients in a series of soil strata having thickness of layers z_1, z_2,..., z_n. (Modified from Capper and Cassie's (1976) equation).


Flow of water
v
v = k1 (dh/dz1)
dz
v = k2 (dh/dz2)
dz
v = kn dh/dzn)
dz
H
Z


1/K
1/k1
1/k2
1/kn
Z
Z1
Z2
Zn

C_s = particle shape factor (varies from 360 for spherical particles to about 700 for angular particles)

D = weighted or characteristic particle diameter

e = void ratio.

The characteristic diameter D is obtained from a grain size analysis using the following equation:

$$D = \frac{\sum M_i}{\sum \frac{M_i}{D_i}}$$

where

M_i = the mass retained between two adjacent sieves.

D_i = the mean diameter of the adjacent sieves.

2) Shallow water pumping test

The volume of water flowing horizontally from a well, in which a constant head of water is maintained by a float valve, is measured. The horizontal permeability determined by this method is a composite rate for the full depth of the hole being tested, but reflects primarily the permeability of the more permeable layers.

A hole is dug by hand to the desired depth. A float apparatus for maintaining a constant head of water in the hole is installed. This float apparatus is connected by tubing to a calibrated supply tank which is on a platform beside the hole. The hole is then filled to the level of the float valve and the water level in the hole is kept constant by means of this valve. The time and the reading on the tank gauge are recorded when everything is operating satisfactorily. Water is added each time the site is visited.

The test should be continued until the material around the hole has become saturated and the flow from the tank is relatively

constant. The permeability should be computed after each visit. When a relatively constant permeability has been reached over a 24 hour period it can be assumed that the periphery of the hole is saturated.

One of the principal limitations of this test is that it requires 2 to 6 days and a considerable amount of equipment. A relatively large amount of water is required also, especially if the material has a permeability above 5 cm per hour (Luthin 1966).

3) The permeameter method

After the proper site has been selected, a hole 1.2 m by 1.2 m is dug to within 7.5 cm of the layer to be tested. The last 7.5 cm are excavated when the equipment is ready to be installed. The equipment consists of an 45.5 cm cylinder. The cylinder is driven 15.25 cm into the soil in the middle of this large hole. About 2.5 cm of clean, uniform, permeable sand is spread over the area inside the cylinder to restrict puddling of the soil surface during the test.

Two 45.5 cm piezometers are driven 22.75 cm below the soil on opposite sides of the cylinder and about 7.5 cm to 10 cm from it. These piezometers are installed by driving 5 cm or 7.5 cm with the driver and then augering out the core, continuing this process until the 22.75 cm mark is at the ground level. A 10 cm cavity is then augered below the piezometer and filled with clean, fine sand. The piezometer is a small diameter pipe driven or jetted into the soil, so that there is no leakage down the outside of the pipe and all entrance of water into the pipe is from the bottom (Israelsen and Maclanghlin, 1935; Wenzel, 1942; Christiansen, 1944; Donnan and Christiansen, 1944)

Two calibrated and tested tensiometers are installed on opposite sides of the cylinder and 7.5 to 10 cm from it on a line at right angles to that of the piezometer. A float valve is installed in the large cylinder to maintain a constant 15 cm head. The float valve is connected to a head tank with 1 cm rubber tubing. When the tensiometers read zero tension, no water shows in the piezometer, and water is moving through the 15 cm test layer at a constant rate, it can be assumed that the requirements of Darcy's law have been met. The flow through the 15 cm test soil cylinder can then be calculated by an application of Darcy's law (Luthin 1966).

4) Pond-Infiltration Test

An infiltration test over a large area has been recommended and put into practice to get away from the soil compression that is inherent in the core samples. The area recommended is 4 m in diameter. The area is surrounded by a circular ditch and filled with water to form a circular pond (a circular pond has less lateral and undesirable seepage loss per unit area than a rectangular one). The pond test procedure is as follows:

Water is added to the pond as needed. When sufficient water has been added to soak the soil down through the layer whose permeability is being analyzed, the falling water level of the pond in the absence of added water is observed. This rate should be a measure of the ability of the soil to pass irrigation and/or drainage water into and through the impermeable layer. Since the flow in this case is almost entirely due to gravity, the hydraulic gradient will be unity and the permeability is calculated from a simple application of Darcy's law that assumes the hydraulic gradient is unity (Luthin 1966).

SECTION TWO

SEDIMENT PROPERTIES INFLUENCING PERMEABILITY

The ease with which water can flow through soils and sediments depends on the properties of the soil. The following are some of these properties which influence permeability:

1) Particle size.
2) Particle shape.
3) Packing.
4) Void ratio.
5) Composition.
6) Degree of saturation.
7) Biological effects.

These properties have an important influence on the coefficient of permeability of soil and sediment formations and are discussed in the following section.

At the end of this section the relationship between water content, degree of saturation and void ratio is outlined.

I) GRAIN SIZE

The size of soil particles range from boulders more than 1m in diameter to clay-size particles less than 0.001 mm in diameter. Table 1 shows the common soil or sediment types (based on grain size) and their approximate particle size range (Folk 1980; Dunn, et al., 1980).

The grain size of a clastic sediment is a measure of the energy of the depositing medium and the energy of the basin of deposition (Reineck and Singh 1980). Clastic sediments consist of particles broken away individually from a parent rock source (Strahler, 1976 p.47). In general, coarser sediments are found in higher-energy environments and finer sediments in low-energy environments. This means that in a river, mean grain size will decrease in a downstream direction. Plumley (1948) showed a decrease in the size of pebbles in the downstream direction of three streams in south Dakota. The fluvial transport of fine sand over several hundred kilometres is incapable of producing a decrease in median and other grain size or mineralogical characteristics (Pollack 1961; Kumar and Singh 1978).

The down-current decrease in grain size is ascribed to two processes - abrasion and progressive sorting (Pettijohn 1957). The latter process, however, in which a decrease in grain size is caused by wear and tear of the grains, gives a more important role to pebbles than to sands. The other process, sorting during transport, is probably the main factor causing the decrease in grain size, especially in sand-size sediments. With a decrease in energy and competency of the transporting medium, coarser sediments are deposited, and only the finer materials transported further. There are other complicating factors. For example tributaries of a main river

Table 1. The Wentworth scale (1922), consists of named sediment classes each having definite upper and lower size class limits.

Wentworth Size Class		Millimeters	Microns	Phi (ϕ)
Gravel	Boulder	1048576	1.04858×10^{9}	-20
		256	2.56×10^{5}	-8
	Cobble			
		64	6.4×10^{4}	-6
	Pebble			
		4	4000	-2
	Granule			
		2.00	2000	-1.0
Sand	Very coarse sand			
		1.00	1000	0.0
	Coarse sand			
		0.50	500	1.0
	Medium sand			
		0.25	250	2.0
	Fine sand			
		0.125	125	3.0
	Very fine sand			
		0.0625	62.5	4.0
Mud	Coarse silt			
		0.031	31	5.0
	Medium silt			
		0.0156	15.6	6.0
	Fine silt			
		0.0078	7.8	7.0
	Very fine silt	0.0039	3.9	8.0
	Clay	0.0020 ↓ 0.00006	2.0 ↓ 0.06	9.0 ↓ 14.0

bringing a different type of sediment, such as coarser sediment, may cause a downstream increase in grain size and decrease in roundness and sphericity. There are also complications in marine shallow-water sediments where the sediment is biologically reworked (Meadows and Tufail 1986). This produces mixed sediments (Reineck and Singh 1980).

The following paragraphs discuss particle size parameters and the effect of particle size on permeability and porosity.

PARTICLE SIZE PARAMETERS

As stated above, sediment particles range from boulders of more than 1m in diameter to clay size particles less than 0.001 mm in diameter (Dunn, et al. 1980). If the sediment is divided into size classes ranged on a logarithmic scale, most sediments have a log-normal size distribution. The size classes show a normal distribution, with a high proportion of particles in the middle class and progressively less towards the extremes.

One of the fundamental purposes in listing sediment parameters is to facilitate the comparision of sediments and to aid in the correlation between sediment types and their environment (Inman, 1952).

It is therefore helpful to classify sediments into several named groups based on the size of the dominant particle size present. Several scales have been devised for this. One of the most frequently used scales is the phi scale, Ø, (Krumbein, 1934). This was introduced as a log transformation to simplify the calculation of sediment characteristics such as mean, median, sorting (standard deviation), skewness and kurtosis (Folk, 1966).

The phi scale is a logarithmic transformation of the size in millimetres. It converts data which is non-normal when measured on a

simple arithmetic scale, to a normal distribution. Because it normalises the data, it makes the data suitable for statistical analysis.

The Wentworth or Ø scale (1922) is now the most commonly used (table 1 page 66). Conversion from millimetres to phi is given by

$$\emptyset = - \log_2 (d)$$

where d = particle diameter in mm.

The size classes at the coarse end of the phi scale are larger in absolute terms than the finer size classes. For example the - 1.0 to - 2.0 Ø range includes particles from 2.00 to 4.00 mm in size, whereas the + 1.0 to + 2.0 Ø range incorporates grains from 0.50 to 0.25 mm in size.

Two methods for determining the particle size parameters are,

I) Graphical analysis.

II) Algebraic analysis.

The following account of the graphical and algebraic analyses is based on the following sedimentological and geological references:

Wentworth (1922).

Krumbein (1934).

Inman (1952).

Folk (1966).

Briggs (1977).

Folk (1980).

Dunn, et al. (1980).

Leeder (1982).

and also on the following mathematical and statistical references:

Kenney and Keeping (1954).

Bevan (1968).

Hays (1969).

Hogg and Craig (1970).

Guttman, et al. (1971).

Lindgren (1976).

Snedecor and Cochran (1980).

Sokal and Rohlf (1981).

Wetherill (1981)

Cohen and Holliday (1982).

Norusis (1983).

Gilchrist (1984).

The mathematical and statistical references have been referred to because much of the mathematics of grain size analyses is based on the analysis of the normal curve in a statistical context.

The references that I have found most useful are:

Inman (1952).

Snedecor and Cochran (1980).

Folk (1980).

Sokal and Rohlf (1981).

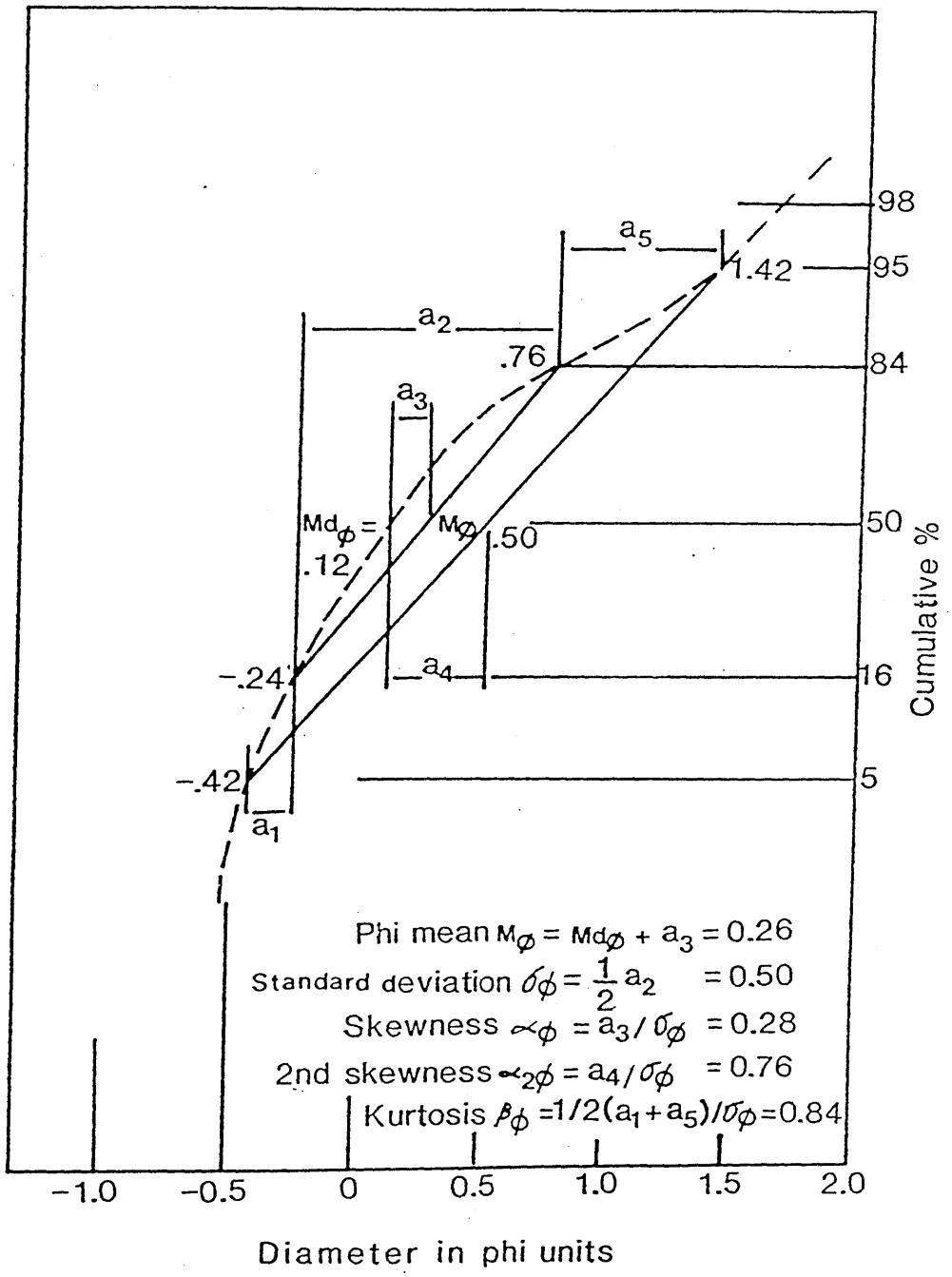
Cohen and Holiday (1982).

Graphical analysis

Size parameters can be calculated directly from the graph of particle size distribution by the use of percentile values A. Percentile value is simply the size value on the X axis corresponding to a selected percentage on the Y axis.

Figure 1 shows a plot of diameter in Ø units against cumulative percentage (weight) for a hypothetical sediment, which describes the parametric data that can be obtained from a graphical analysis. The following parameters can be obtained by using 95%, 50%, 16% and 5%

Figure 1. A plot of diameter in Ø units against cumulative percentage (weight) for a representative sediment, which describes the parametric data that can be obtained from this type of curve (Inman, 1952).


98
95
84
50
16
5
Cumulative %
a5
1.42
a2
.76
a3
Md$_\phi$ =
.12
M$_\phi$
.50
-.24
a4
-.42
a1
Phi mean M$_\phi$ = Md$_\phi$ + a$_3$ = 0.26
Standard deviation $\sigma_\phi = \frac{1}{2} a_2$ = 0.50
Skewness α_ϕ = a$_3$ / σ_ϕ = 0.28
2nd skewness $\alpha_{2\phi}$ = a$_4$ / σ_ϕ = 0.76
Kurtosis β_ϕ = 1/2(a$_1$ + a$_5$)/σ_ϕ = 0.84
-1.0
-0.5
0
0.5
1.0
1.5
2.0
Diameter in phi units

percentiles: phi mean diameter $M_{\emptyset}$

Phi median diameter $Md_{\emptyset}$

standard deviation $\sigma_{\emptyset}$

skewness $\alpha_{\emptyset}$

second skewness $\alpha_{2\emptyset}$

kurtosis $\beta_{\emptyset}$

These parameters are defined and calculated as follows.

Phi mean diameter

The phi mean diameter is the mean particle size in phi units. It is taken as the average of the 16th and 84th percentile diameters as follows

$$M_{\emptyset} = \left(\frac{1}{2}\right)(\emptyset_{16} + \emptyset_{84}) = Md_{\emptyset} + (\sigma_{\emptyset}\alpha_{\emptyset})$$

where

$\sigma\emptyset$ = phi deviation measure

$\alpha_{\emptyset}$ = phi skewness measure.

These are discussed in later paragraphs.

Phi median diameter

The phi median diameter is the diameter value of the ordinate that divides the frequency distribution curve of a sediment's particle size into two equal areas. It is defined as

$$Md_{\emptyset} = \emptyset_{50} = M_{\emptyset} - (\sigma_{\emptyset}\alpha_{\emptyset})$$

The median is less affected by extreme values of skewness than the mean because it is closer to the modal diameter* than the mean.

---

*The mode refers to the largest value of the variable in a frequency distribution, or the value represented by the greatest number of individuals. Distributions having two peaks (equal or unequal in height) are called bi-modal; those with more than two peaks are multimodal. In unimodal, symmetrical distributions the mean, the median, and the mode are all identical. In a typical symetrical distribution, the relative positions of the mode, median, and mean are generally these: the mean is closest to the drawn-out tail of the distribution, the mode is farthest, and the median is in between these (Sokal and Rohlf, 1981, pp.46-47).

Standard deviation

The standard deviation of a sediment's size frequency distribution in the phi notation can be approximated graphically by measuring the distance between the 16th and 84th percentile diameters on a cumulative frequency curve. The standard deviation is then given as:

$$\sigma_{\phi} = \left(\frac{1}{2}\right) (\phi_{84} - \phi_{16})$$

Skewness

Skewness is a type of departure from normality often seen in sediment samples, in which one tail of the curve is drawn out more than the other. In such curves the mean and the median will not coincide. Curves are skewed to the right or left. If they are skewed to the right the tail is towards larger values, and the distribution is termed positively skewed (figure 2(i)). If they are skewed to the left, the tail is towards smaller values, and the distribution is termed negatively skewed (figure 2(ii)). Values of skewness are zero if the observed distribution is exactly normal.

Skewness is measured as primary and secondary skewness.

1) The primary skewness measure α_{ϕ} is the amount of the departure of the distribution from the normal and it is sensitive to skew properties occurring in the bulk of the grain size distribution. It is defined as:

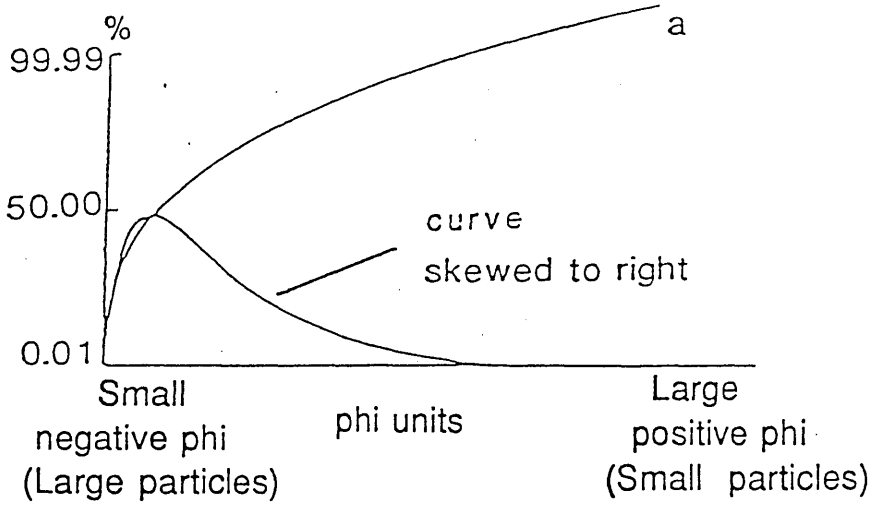
$$\alpha_{\phi} = \frac{M_{\phi} - Md_{\phi}}{\sigma_{\phi}}$$

If the distribution is skewed towards smaller phi values (coarser diameter), the phi mean is numerically less than the median and the skewness is negative. Conversely, α_{ϕ} is positive for distributions skewed towards higher phi values.

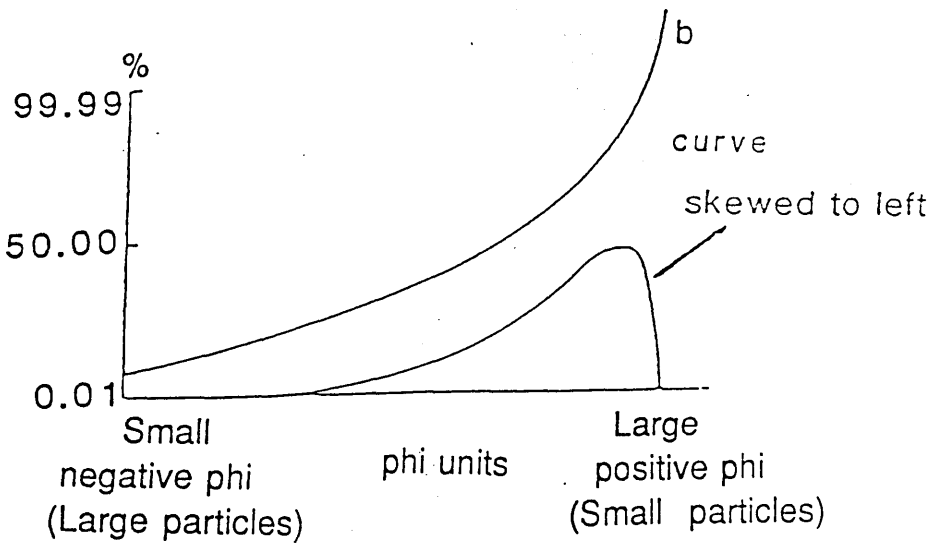
Figure 2(i). An example of frequency distribution plotted with the ordinate in normal probability scale. The curve with the long tail to the right is called skewed to the right or positively skewed. Curve a is called the cumulative curve (Sokal and Rohlf 1981).

Figure 2(ii). An example of frequency distribution plotted with the ordinate in normal probability scale. The curve with the long tail to the left is called skewed to the left or negatively skewed. Curve b represent the cumulative curve (Sokal and Rohlf 1981).

2(i)


%
99.99
50.00
0.01
a
curve
skewed to right
Small
negative phi
(Large particles)
phi units
Large
positive phi
(Small particles)

2(ii)


%
99.99
50.00
0.01
b
curve
skewed to left
Small
negative phi
(Large particles)
phi units
Large
positive phi
(Small particles)

The mean of the distribution is given by the intersection of a straight line drawn between ϕ_{16} and ϕ_{84} with the 50 percent line (see fig. 1 page 70). If this point is greater than the median, the curve has a positive skewness.

It should be noted that the straight line drawn between the 16th and 84th percentiles on probability paper, represents a normal distribution having the same mean and standard deviation as the distribution under consideration.

2) The secondary measure of skewness, called the second phi skewness measure, $\alpha_{2\phi}$. This measure is sensitive to skew properties occuring in the tails of the distribution (very coarse or very fine particle sizes). It is based on the 5th and 95th percentile diameters. It is defined as

$$\alpha_{2\phi} = \frac{\left(\frac{1}{2}\right)(\phi_5 + \phi_{95}) - Md_\phi}{\sigma_\phi}$$

The significance of this measure can be illustrated graphically by constructing a straight line between ϕ_5 and ϕ_{95} (see fig 1 page 70). The straight line in this case represents a normal distribution having the same spread between the 5th and 95th percentile diameters as the curve under consideration.

Kurtosis

Kurtosis is the other type of departure from normality sometimes seen in the size distribution of sediment samples, and statistically is related to the fourth moment about the mean $\left(\frac{\sum (X_i - X)^4}{n - 1}\right)$.

The value of kurtosis is zero if the observed distribution is exactly normal. If the value of kurtosis is greater than zero

(positive), the distribution has a higher central peak falling rapidly on either side of the mean to longer tails, when compared to a normal distribution having the same standard deviation. This is called leptokurtosis (figure 3(i)).

When the value of kurtosis is less than zero (negative), the distribution has a lower central peak (is flatter), and tends to be convex with little or no tails at the extremes of the distribution, when compared to a normal distribution having the same standard deviation. This is called platykurtosis (figure 3(ii)).

The phi kurtosis measure is a dimensionless measure of the average spread between the percentile diameter ϕ_5, ϕ_{16} and ϕ_{84}, ϕ_{95}, and is shown graphically in figure 1 as (1/2) $(a_1 + a_5)$ divided by the standard deviation.

$$\beta_\phi = \frac{\frac{1}{2}(\phi_{16} - \phi_5) + \frac{1}{2}(\phi_{95} - \phi_{84})}{\sigma_\phi}$$

$$\beta_\phi = \frac{\frac{1}{2}(\phi_{95} - \phi_5) - r_\phi}{\sigma_\phi}$$

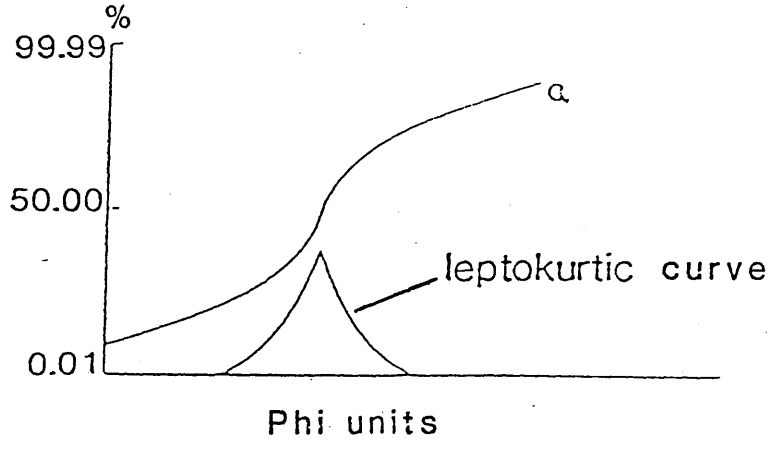
$$\beta_\phi = \frac{\frac{1}{2}(a_1 + a_5)}{\sigma_\phi}$$

Inman (1952, p.138) calculates the value of kurtosis as 0.65 in the normal distribution using a graphical method. The value greater than 0.65 gives a leptokurtic curve (positive kurtosis) and the value less than 0.65 gives a platykurtic curve (negative kurtosis). This calculation has been corrected by Meadows and Mgherbi (personal communication).

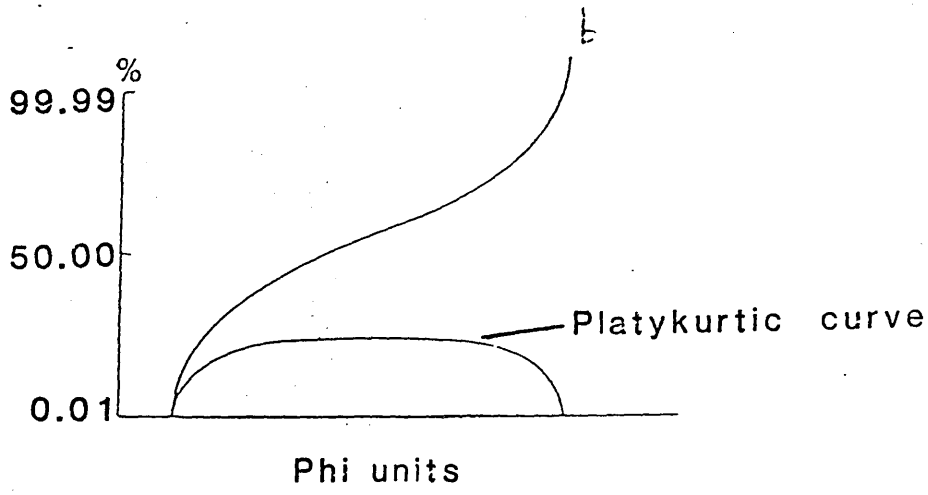
Figure 3(i). Leptokurtic curve. Frequency distribution plotted with the ordinate in a normal probability scale. Curve a represents the cumulative curve (Sokal and Rohlf 1981).

Figure 3(ii). Platykurtic curve. Frequency distribution plotted with the ordinate in a normal probability scale. Curve b represents the cumulative curve (Sokal and Rohlf 1981).

3(i)


%
99.99
50.00
0.01
a
leptokurtic curve
Phi units

3(ii)


%
99.99
50.00
0.01
b
Platykurtic curve
Phi units

The arithmetic mean

The arithmetic mean is called the mean or the average. The mean of grain size sample $\bar{X}$ is calculated by adding all the individual observations of X_i, where X_i is the diameter of the ith particle, in a given sample n and dividing this sum by the number of items (sand grains) in the sample. Algebraically, $\bar{X}$, is expressed as

$$\bar{X} = \frac{\sum X_i}{n} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

where $X_i = x_1, x_2, \ldots, x_n$.

The median

The median grain size is defined algebraically in a similar manner to the graphical definition as that value of the individual observations of X_i, where X_i is the diameter of the particle, that has an equal number of items (sand grains) on either side of it. Thus, the median divides a frequency distribution into two halves. With an odd number of individuals X_i it is easy to find the median. When the number in the sample is even, the median is conventionally calculated as the midpoint between two varieties.

The standard deviation

The standard deviation (σ) of sediment size is a measure of the dispersion of probability about the highest point and it is the square root of the variance. The standard deviation is often called sorting in sedimentology.

The variance (σ^2) is the sum of the deviation squares of all observations of X_i, where X_i is the diameter of the ith particle, from their mean and is the second moment about the mean.

$$\sigma = \sqrt{\frac{1}{n-1} \sum (X_i - \bar{X})^2}$$

$$\sigma^2 = \frac{1}{n-1} \sum (X_i - \bar{X})^2$$

where $X_i = x_1, x_2, \ldots, x_n$.

The reason for the division by (n - 1) is that with smaller n's, (n - 1) gives a less-biased estimate of the variance of the population from which the particular sample is drawn (Hamburg 1974 [p.64]; Snedecor and Cochran 1980; Sokal and Rohlf 1981 [p.53]; Cohen and Holliday 1982).

Skewness

The third moment about the mean of a sample from a population is

$$\frac{\sum (X_i - \bar{X})^3}{n - 1}$$

The sample statistic for measuring skewness of a sediment's size distribution algebraically is the third moment about the mean of the normal distribution divided by the cube of the standard deviation

$$\text{skewness} = \frac{\sum (X_i - \bar{X})^3}{(n - 1)\sigma^3}$$

where $X_i = x_1, x_2, \ldots, x_n$.

Pictorial representations of negatively and positively skewed distributions and their cumulative plots on probability paper are shown in figure 2(i) and 2(ii).

Kurtosis

The fourth moment about the mean of a sample from a population is:

$$\frac{\sum (X_i - \bar{X})4}{n - 1}$$

where $X_i = x_1, x_2, \ldots, x_n$.

The coefficient of kurtosis is the fourth moment about the mean minus 3 divided by the fourth power of the standard deviation:

$$\text{kurtosis} = \frac{\sum (Xi - \bar{X})^4}{(n - 1)\,\sigma^4} - 3$$

Note: x1, x2,..., xn indicate the weight of particles retained on each grade sieve, which is expressed as a percentage of weight on each sieve.

The effect of sorting on permeability

Permeability increases with increasing sorting. It has been found that the finer the grain size and the poorer the sorting of sand, the smaller its permeability (Krumbein and Monk 1942; Pettijohn, 1976).

The effect of grain size on permeability

A coarse grained sediment such as gravel is generally more permeable than a fine-grained sediment such as fine sand (Krumbein and Monk, 1942; Wallace, 1948; Pettijohn, et al. 1984). A greater resistance to water flow is offered by the much smaller voids associated with the fine-grained sediment. However, if a considerable amount of fine sand is added to the gravel its permeability decreases due to a reduction in void size caused by filling of the voids with sand (Fraser 1935).

The effect of particle size on porosity

The actual particle size is theoretically immaterial. However all ordinary depositional mechanisms are such that the coarser the average grain size the greater the overall variety of sizes. A rock may easily consist of very fine sand grains and little else; it would not survive long consisting of tennis-ball-sized cobbles and nothing else. Hence finer-grained sediments in general have higher porosities than

coarser-grained sediments because there are invariably other factors in play. For example, freshly deposited clays have porosities of 50-85 percent. Fine sandy loam may attain 52 percent, and fine sand 48 percent, but coarse sand without cement is unlikely to surpass 40 percent (Trask, 1931; Fraser, 1935; Friedman and Sanders, 1978; North, 1985).

II) GRAIN SHAPE

The following section contains an introductory account of grain shape, its fundamental shape properties of sedimentary particles (roundness and sphericity) and the effect of grain shape on permeability. The effect of grain shape on porosity will also be discussed, particulary in relation to Fraser's experiment (1935).

Grain shape in sediments may never be truly spherical, and varies within wide limits in relation to the following factors (Fraser, 1935 pp.934).

1) Character of the original mineral or material.
2) Shape of the original fragments.
3) Degree of wear that grains have undergone.
4) Method of transportation and deposition.

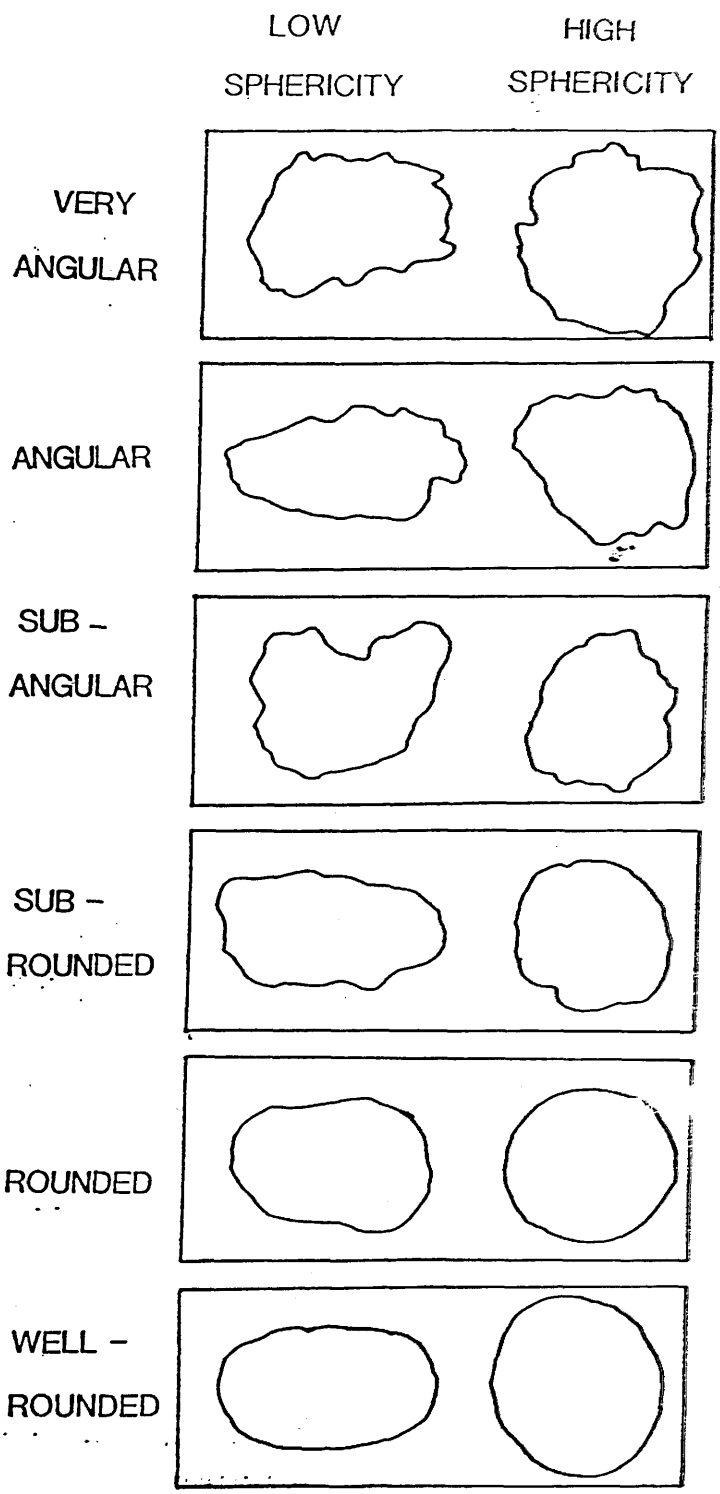
The two fundamental grain shape properties of sedimentary particles that are usually measured are:

1) Roundness
2) Sphericity.

Roundness and sphericity are often confused with each other. The difference can be visually appreciated by inspecting figure 4 (Powers, 1953). The definition of roundness and sphericity are as follows:

1) Roundness is an attribute of form of a particle that is related to the sharpness or curvature of edges and of corners (Friedman, and Sanders, 1978).
2) Sphericity is an attribute of form which defines how nearly equal are the three dimensions; long, intermediate and short diameter of a particle (Folk, 1980).

Figure 4. The difference between roundness (6 rows) and sphericity (2 columns) (Powers 1953).


LOW
SPHERICITY
HIGH
SPHERICITY
VERY
ANGULAR
ANGULAR
SUB –
ANGULAR
SUB –
ROUNDED
ROUNDED
WELL –
ROUNDED

A solid may have a maximum degree of roundness and still not be a sphere (i.e. still not be perfectly spherical). A cylinder terminated at each end by a half sphere, (figure 5) is completely rounded, and yet the cylinder is not a sphere (i.e. is not spherical). A cylinder of this shape may well be worn down to a sphere, where upon it will of course also be totally spherical as well as being completely round. Conversely, a solid may have a high degree of sphericity and little roundness. An example is the dodecahedral form of garnet. A dodecahedron is a three dimensional shape having twelve faces. Each face is pentagonal. If not worn, it exhibits non-rounded sharp corners with large obtuse angles. The shape of the dodecahedron is spherical in an overall sense, but it has sharp corners and therefore has little roundness.

Roundness and sphericity have a close relationship to grain volume. However roundness is likely to have a greater effect than sphericity (Al-hamdan, 1975).

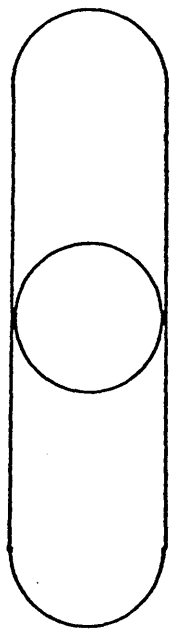
I shall now deal with roundness and sphericity and the methods of determining them in some detail.

2.1 Roundness

Before defining how roundness is measured, it is important to understand how a corner is defined since the concept of a corner is critical to the concept of roundness.

A corner may be defined as every such part of the outline of a projection area which has a radius of curvature equal to or less than the radius of curvature of the maximum inscribed circle of the same area. A corner has reached its maximum degree of roundness when the radius of its curvature equals the radius of the maximum inscribed

Figure 5. A cylinder terminated at each end by a half sphere is completely rounded, and yet the cylinder is not spherical (Wadell, 1932).



circle. An increase of its radius of curvature over that of the inscribed circle results in one, two, or several new corners of smaller radii of curvature, i.e., less degree of roundness (figure 6), (Wadell, 1932).

The degree of rounding generally varies between different grain sizes in any natural deposit, because of differences in the mineralogical composition. Marshall (1929) found that variation in rounding of various grain sizes on a beach, where no pebbles had a diameter larger than 6.7 mm, was as follows:

1) All material coarser than 3.4 mm, well rounded.
2) Between 3.4 and 0.84 mm, mostly angular.
3) Between 0.84 and 0.42 mm, quite angular.
4) From 0.42 to 0.25 mm, fairly well rounded.
5) All grades finer than 0.25 mm, composed of well rounded grains.

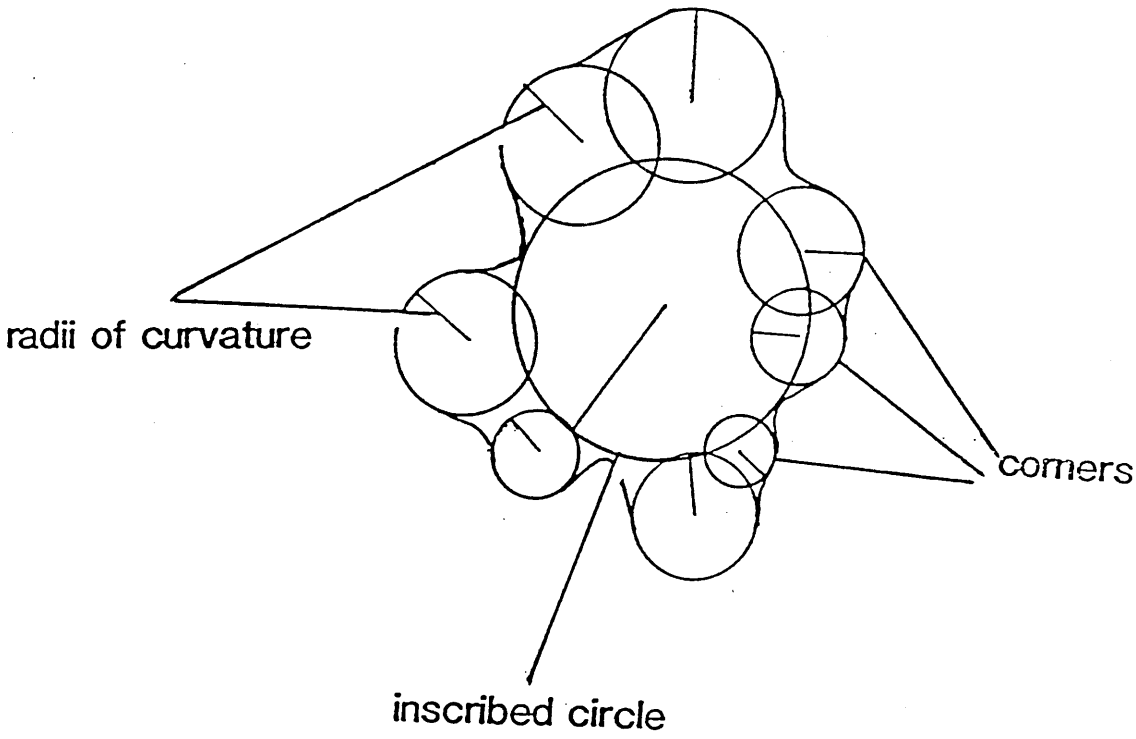
He found that the rounded form of the coarser material was due to simple abrasion while the angular form of the intermediate sizes was the result of impact. He showed experimentally that impact acted on intermediate particles more rapidly than did abrasion. The rounded form of the smaller sizes was caused by grinding, which is more important than impact when the grains are small.

Methods of determining the roundness of grains

There are many methods of determines the roundness of grains:

1) Trowbridge and Mortimore (1925) were the first people to use a visual comparision of grains to find the roundness, but the grain shapes which they chose were hypothetical.
2) Wadell (1932,1933,1935) defined roundness as being related

Figure 6. The definition of a corner which is every such part of the out line of a projection area which has a radius of curvature equal to or less than the radius of curvature of the maximum inscribed circle of the same area (Wadell 1932).


radii of curvature
corners
inscribed circle

to the acuteness or smoothness of the angles and corners of grains.

3) Russel and Taylor (1937) suggested five groups of roundness which were also determined visually.

4) Krumbein (1941) suggested a method of visual comparision, where nine classes were differentiated.

5) Powers (1953) suggested six classes of roundness (well rounded, rounded, sub rounded, sub angular, angular and very angular).

6) Robson (1958), suggested a new method of finding roundness using a microscope with a graticule.

7) Boggs (1967) used a modern technique involving electronic machinery normally used in the analysis of particle size. He modified this to measure the roundness and sphericity of grains. This proved an accurate method and is useful for measuring large numbers of grains.

I have chosen two methods of measuring roundness to describe. The first one is Wadell's method (1932) and the second is Boggs's method (1967).

Boggs's method is a more modern technique which uses electronic machinery to measure the roundness and sphericity. It also involves using Wadell's equation, and so Wadell's method has also been included.

1) Wadell's method

The total roundness of a solid is obtained by measurements in three planes at right angles to each other, but two planes are in most cases sufficient, while one plane is satisfactory when dealing with small sedimentary particles.

The principle of this method is based on taking measurements of the radius of the maximum inscribed circle and also of the radii of curvature of the corners. These values are then substituted in the equation below.

The curvature of the corners of a sand grain may easily be measured if submitted to sufficient magnification. The values obtained, however, would not be directly comparable with the same kind of values of a boulder. Large objects such as boulders and cobbles must be reduced, and small ones like sand grains magnified to approximately the same size, i.e., the standard size, on which the measurements are performed. The average diameter of the standard size used for measurement of particles has been fixed at 7cm, and microscopic particles have been enlarged to about that size by camera lucida or screen projection.

Since the radius of curvature of a corner may attain any value up to the maximum which equals the value of the maximum inscribed circle, the roundness of a corner may be expressed by the ratio $\left(\frac{r}{R}\right)$, where,

r = the radius of curvature of the corner.

R = the radius of the maximum inscribed circle.

The maximum value for $\left(\frac{r}{R}\right)$ is 1.0 for a corner of maximum roundness. The total roundness of a solid in one plane may be obtained by taking the average of the roundness of the individual corners in that plane. The formula for roundness is as follows:

$$\text{Degree of roundness of a particle} = \frac{\sum (r/R)}{N} = \frac{\sum r}{RN} \qquad (1)$$

where,

$\sum (r/R)$ = the sum of the roundness values of the corners.

N = the number of corners of the particle.

The maximum value for roundness achieved by this formula is 1 for

a particle of maximum roundness in a given plane. It is important to note that a sphere always has maximum roundness, but non-spherical smooth shapes can also have maximum roundness (figure 4 and 5 pp. 86 and 89). The division by R in formula (1) (the radius of the maximum inscribed circle) is necessary for the following reason.

Consider two different sized particles with the same shape. Let the first particle (a) have four corners with four values of radius of curvature 2,3,4, and 5 respectively. Let the value of the radius of the maximum inscribed circle be 10. Let the second particle (b) have the same shape as the first but be ten times larger. The radii of curvature of the corners are 20, 30, 40 and 50 respectively and the radius of the maximum inscribed circle is 100. Table (2) shows the difference between the formula with R and the other formula without division by R. As mentioned above, because the two grains have the same shape they must give the same degree of roundness. The formula which includes division by R gives the same degree of roundness for both particles. However, the formula which doesn't include division by R gives a different degree of roundness.

A formula which gives a slightly different roundness value from $\frac{\sum (r/R)}{N}$ has been used to measure the roundness of grains. It is

$$\frac{N}{\sum (R/r)} \qquad (2)$$

The maximum value achieved by this formula is also 1 for a solid of a maximum roundness.

Wadell (1933, 1935) indicated that he preferred the second formula (formula 2), as being more useful because it gives a lower value for the roundness of particles. Relatively well-rounded particles which

grain	radius of curvature	R present (r/R)/N	R absent r/N
a	2 3 4 5	0.35	3.5
b	20 30 40 50	0.35	35

Table 2. The difference between the formula for the degree of roundness when R is included (as is usually done) and when R is not included.

have obtained a very low degree of roundness for one or more corners by chipping or fracturing shortly before deposition generally have a lower roundness value using formula (2) than using formula (1). These particles of lower value have corners of greatly different roundness value, i.e. when the coefficient of variation of the roundness value is high*(Wadell,1935).

2)Boggs's method

The Zeiss TGZ3 particle size analyzer is a semi-automatic instrument designed to permit rapid size measurement of large numbers of grains (Figure 7). Although the TGZ3 analyzer was designed primarily for particle size analysis, it lends itself readily to measurement of roundness parameters (radius of curvature of the inscribed circle, radii of curvature of corners) and also to sphericity parameters (radii of curvature diameters of inscribed and circumscribed circle -see below). The procedure is as follows.

Loose grains to be photographed are placed on a slide. The slide is tapped gently so that they come to rest with long and intermediate axes in the projection view (i.e. as seen from above). The grains are photographed, and a suitably enlarged photomicrograph is prepared on thin photographic paper. The measuring mark (a circular light spot) can be adjusted through a range extending from 1.2 mm to 27.7 mm. The photograph is placed on the photomicrograph support and a grain selected for measurement. Starting at a given corner of the grain, the light spot is adjusted by means of the handwheel to the appropriate

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* The coefficient of variation is the standard deviation as percentage of the arithmetic mean, thus: $C_v = (\sigma/\mu)x(100)$,
where C_v is the coefficient of variation, σ the standard deviation, and μ the arithmetic mean of a given number of variables (Snedecor and Cochran, 1980, p.37).

Figure 7. Zeiss TGZ3 particle size analyzer is a semi-automatic instrument designed to permit rapid size measurement of large numbers of grains (Boggs, 1967).

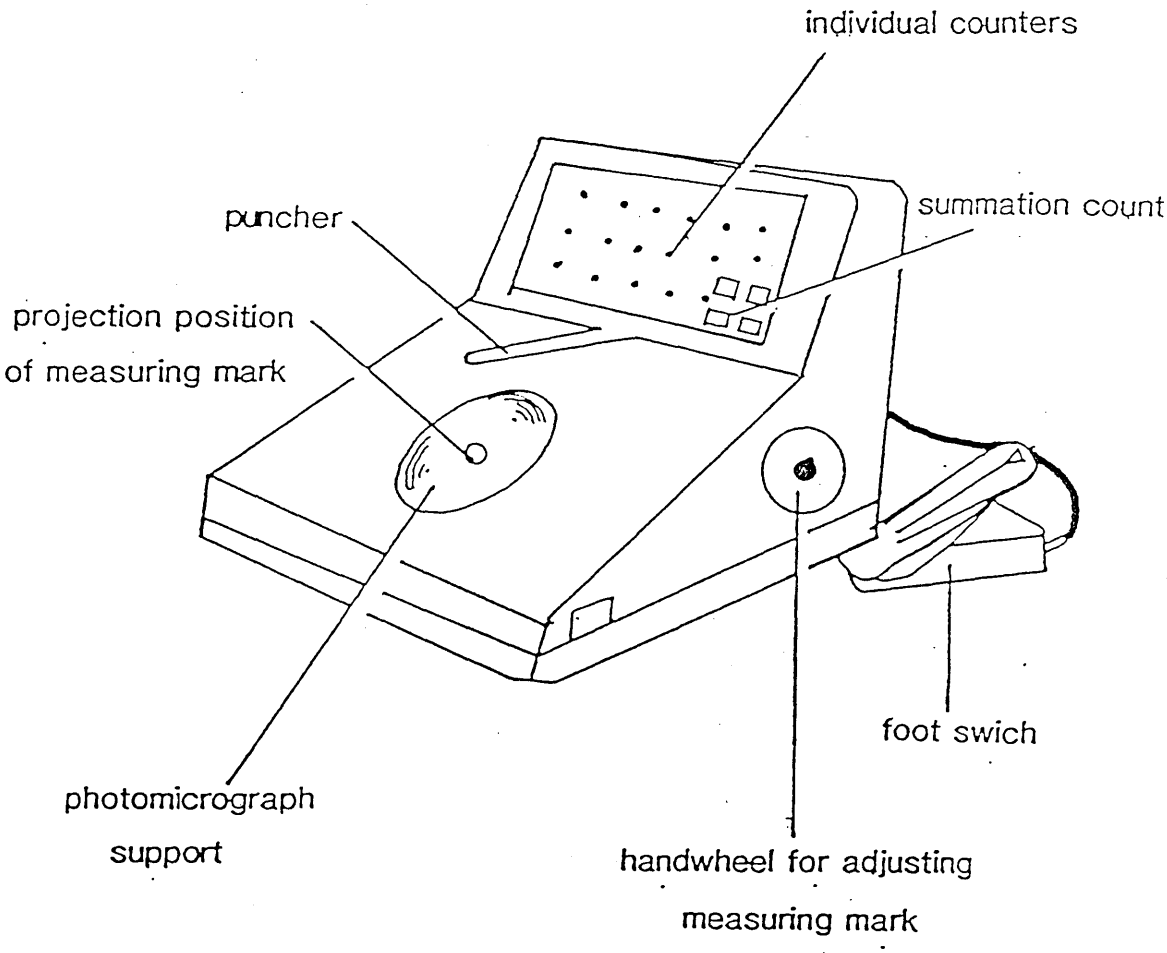
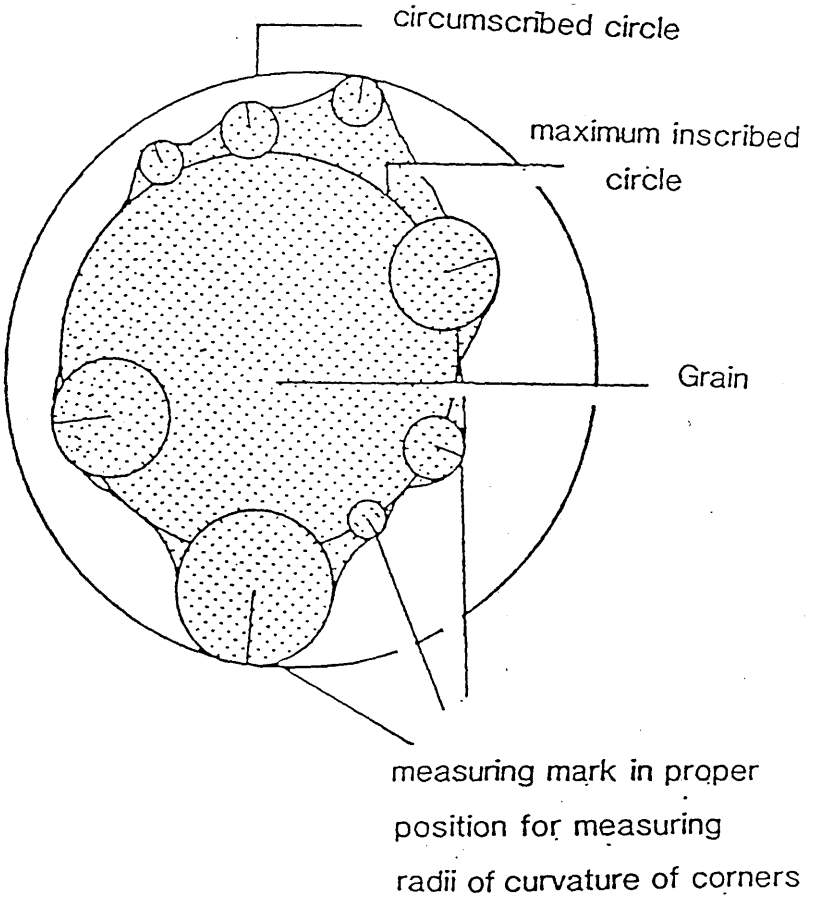

individual counters
puncher
summation count
projection position
of measuring mark
foot swich
photomicrograph
support
handwheel for adjusting
measuring mark

Figure 8. Diagram of enlarged grain showing measuring mark (light spot) in position for measuring radii of curvature of corners, and diameters of circumscribed and inscribed circles (Boggs 1967, page 912). The shaded area represent the sand grain, and the circles represent the various positions of the light spot, when measuring the inscribed and circumscribed circles, and the radii of curvature of each of the corners.


circumscribed circle
maximum inscribed
circle
Grain
measuring mark in proper
position for measuring
radii of curvature of corners

diameter to fit the curvature of the corner (figure 8). The foot switch is depressed, recording the diameter of the light spot (and thus the diameter of curvature of the corner) in the appropriate corner and simultaneously punching a hole in the grain corner just measured. The photograph is then shifted slightly to bring the next corner of the grain into measuring position and the process is repeated. This continues until all corners are measured. The measuring mark is next adjusted to measure the diameter of the maximum inscribed circle and the diameter of the circumscribed circle. The data from the counters are then recorded on a data sheet, the counter panel is cleared and the procedure repeated on the next grain. Then, the roundness can be measured using Wadell's roundness equation and also the sphericity of the grain (see below) (Boggs, 1967).

2.2 Sphericity

The attribute of form known as sphericity defines the degree to which a particle approximates the shape of a sphere; it expresses how nearly equal are the three mutually perpendicular dimensions of a particle. Comparisions of particles with spheres can be made on the basis of surface areas, volumes, and ratios of lengths of axes or diameters.

Methods of determining the sphericity of grains

There are many methods of determining the sphericity of grains.

1) Wadell (1932, 1933, 1935) defined the sphericity as the ratio of the surface area of a sphere of the same volume as the particle to the actual surface area of the particle (Winterkorn and Fang, 1975). At a later date Wadell gave another simpler formula to define the sphericity as the ratio of the diameter of the inscribed circle to the diameter of the

circumscribed circle of the particle.

2) Zingg (1935) studied the shape of grains and divided shape into four classes. These classes were defined by the relationship between the long, intermediate, and short dimensions of the grains.

3) Riley (1941) proposed an expression of sphericity based the diameters of two circles. He called this the projection sphericity and defined it as the square root of the ratio of the inscribed and circumscribed circles:

$$\text{Projection sphericity} = \sqrt{\frac{D_i}{D_c}}$$

where,

D_i = the diameter of the inscribed circle.
D_c = the diameter of the circumscribed circle.

4) Krumbein (1941) expressed the volume of a particle in terms of an ellipsoid* (see page 109), (refered to in Krumbein (1941) as

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*A quadric surface is any locus in three-dimensional space which can be represented in a given coordinate system by a quadratic (polynomial) equation, using the variables x, y, and z. The equation takes the form

$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$ (1)

where a, b, c, d, f, g, h, u, v, and w are real constants with a, b, c, f, g and h not all zero. By translation and rotation of axes, equation (1), when it represents a non-empty locus, can be reduced to one of the canonical forms which are ellipsoid, hyperboloid, and paraboloid.

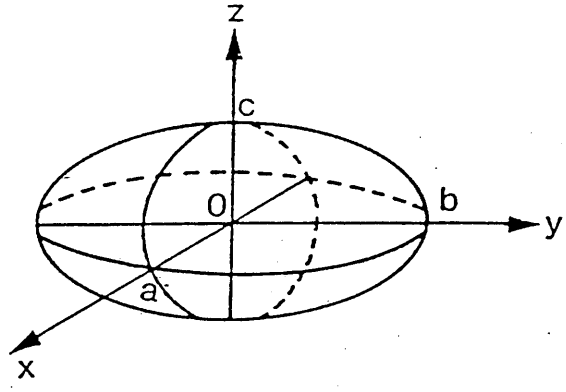
In this paragraph only ellipsoid surfaces will be defined, i.e. the locus of points p(x,y,z) in three-dimensional space. The equation of the ellipsoid is $(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1$.
All the non-trivial (at least one of the variables should not be equal to zero) plane sections are ellipses (see below) or circles. The x, y, and z axes are called the principal axes of the ellipsoid, the surface having the coordinate planes as planes of symmetry (figure, 9a) (Hunter,1972).

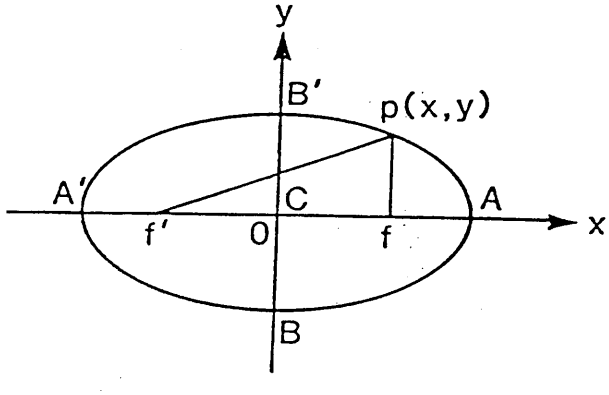
Ellipse

An ellipse is the locus of points P(x,y) in two dimensional space the sum of whose distances from two fixed points is constant (Thomas, 1973). The equation for an ellipse is $(x^2/a^2) + (y^2/b^2) = 1$.
In figure 9b, A, A' are called the vertices of the ellipse; A'A is called the major axis and its length A'A is denoted by 2a; B'B is called the minor axis of the ellipse and its length is denoted by 2b. The midpoint c of A'A is called the centre of the ellipse (figure, 9b) (Hunter, 1972).

Figure 9a. Diagram of an ellipsoid shows the surface, and the three axes (x, y, and z) of the ellipsoid. b, a and c are the semimajor axis, semimiddle axis and semiminor axis respectively (Hunter, 1972).

Figure 9b. Diagram of an ellipse shows the vertices, and major, and minor axes of the ellipse. A, A' are called the vertices of the ellipse, A'A is called the major axis; B'B is called the minor axis of the ellipse. The midpoint c of A'A is called the centre of the ellipse (Hunter, 1972).


z
c
0
b
y
a
x


y
B'
p(x,y)
A'
C
A
x
f'
0
f
B

a triaxial ellipsoid) having three diameters L, I, and S where L, I, and S are long, intermediate and short axes respectively. Krumbein defined the intercept sphericity as

$$\text{The intercept sphericity} = \frac{IS}{L^2}$$

5) Folk (1946 unpublished referred to in Sneed and Folk, 1958, p.118) defined a new sphericity value called the maximum projection sphericity which is expressed as

$$\text{Maximum projection sphericity} = \frac{S^2}{LI}$$

In the follow paragraphs I describe in detail Wadell's and Folk's methods for measuring sphericity. There are two reasons for choosing these methods:

1) Most studies refer to Wadell's equation because it is the basis for measuring sphericity.

2) Folk (1946- see above) used two methods to determine sphericity. One uses an equation and the other uses a triangular graph.

1) Wadell's method

Wadell (1933) noted that there should be some kind of relationship between volume, surface, and sedimentological shape. He referred to Steiner's thesis in Weierstrass (1881,1882) "Unter allen Korpern von gleichem Inhalte hat die Kugel die Kleinste Oberflache; und unter allen Korpern von gleicher Oberflache hat diesselbe den grossten Inhalt." which can be translated as "Of all bodies of the same volume

the sphere has the least surface; and of all bodies of the same surface it has the greatest volume.". These relations together with Steiner's thesis, are the basis for the following:

1) Solids of equal surface area and equal volumes have the same shape.

2) Solids of equal surface area but of different volumes have different shapes.

3) Solids of equal volumes but of different surface area have different shapes.

These are summarised in table (3).

Wadell (1933) chose to develop his equation using solids of equal volumes but different surface areas, because they were more convenient for sedimentological purposes (number (3) in table (3)). The numerical value of the shape-character of a particle was then expressed with reference to a sphere by the ratio of the surface area of a sphere of the same volume as the particle, to the actual surface area of the particle :

$$\text{Degree of true sphericity} = \frac{s}{S} \qquad (3)$$

where

s = the surface area of a sphere of the same volume as the particle.

S = the actual surface area of the particle.

The value obtained is independent of the size of the particle, and thus is dimensionless.

The maximum value obtained by the formula $(\frac{s}{S})$ is 1 - which is the numerical value for a sphere. Because of the difficulty of measuring the surface area of an irregular solid, Wadell (1935) used another estimate of sphericity which he called the projection

	surface area	volume	shape
1	equal	equal	equal
2	equal	different	different
3	different	equal	different

Table 3. Composition of surface area, volume, and shape for two solid objects.

sphericity:

$$\sqrt[3]{\frac{V_1}{V_2}} \qquad (4) \qquad \text{or} \qquad \frac{D_1}{D_2} \qquad (5)$$

where

V_1 = the volume of the particle.

V_2 = the volume of the smallest sphere that encloses the particle.

D_1 = the diameter of a sphere of the same volume as the particle (nominal diameter).

D_2 = the diameter of the circumscribing sphere.

Equation (5) is derived from equation (4) as follows: Wadell expressed the volume of the particle in terms of a sphere having the same volume.

The general formula for the volume of a sphere is

$$\frac{4}{3} (\pi) r^3 \text{ , where r = radius,}$$

The diameter of the corresponding sphere is D_1 , therefore the volume of the corresponding sphere is

$$\frac{4}{3} (\pi) r^3 = \frac{4}{3} (\pi) \left(\frac{D_1}{2}\right)^3 = \frac{\pi}{6} (D_1)^3 .$$

Wadell chose solids of equal volume but different surface area, so the volume of the corresponding sphere is equal to the volume of the particle,

$$V_1 = \frac{\pi}{6} (D_1)^3$$

where V_1 is the volume of the particle.

The volume of the circumscribed sphere is in general based on the longest diameter D_2 of the particle so that the volume of this sphere is

$$V_2 = \left(\frac{\pi}{6}\right) (D_2)^3$$

By substituting these values in equation (4), Wadell's sphericity reduced formula (4) to the ratio of the nominal diameter of the particle to its longest diameter

$$\sqrt[3]{\frac{V_1}{V_2}} = \left[\frac{V_1}{V_2}\right]^{1/3} = \left[\frac{\left(\frac{\pi}{6}\right)(D_1)^3}{\left(\frac{\pi}{6}\right)(D_2)^3}\right]^{1/3}$$

then

$$\sqrt[3]{\frac{V_1}{V_2}} = \frac{D_1}{D_2}$$

2) Folk's methods

Folk (1946) measured the three axes of each grain by vernier calipers (A short scale sliding on a graduated scale to give fractional readings) according to the method suggested by Krumbein (1941, p.65-66). Krumbein modified Wadell's formula (1934) for use with calipers by assuming that the particle was a triaxial ellipsoid (figure 9a,b) with the axes a, b,and c (equivalent to L, I, and S) and substituting this value for the true volume.

Folk (1946) suggested that a more natural measure of sphericity can be developed to take into account the actual hydraulic behaviour of a particle. Assuming constant density, temperature, and gravity, two factors affect the settling velocity of particles in water: the

volume, which affects the downward force, and the surface area opposed to the direction of motion, which provides an upward resistive force. By comparing the volume of the particle with its maximum projection area, Folk defined the maximum projection sphericity as

"The maximum projection area of a sphere of the same volume as the particle divided by the maximum projection area of the particle". (6)

The derivation is as follows, For the purposes of this formula, the particle is assumed to approximate a triaxial ellipsoid with axes L, I, and S (long, intermediate, and short, respectively).

The general formula of the area of an ellipse with two axes is

$$\pi \times \text{semimajor axis} \times \text{semiminor axis}$$

where the semimajor and semiminor axes are $\frac{L}{2}$ and $\frac{S}{2}$ respectively.

As only two axes L and I for maximum projection area are apparent in an ellipsoid with three axes, the maximum projection area of the ellipsoid (with three axes) is

$$\pi \times \left(\frac{L}{2}\right) \times \left(\frac{I}{2}\right) = \left(\frac{\pi}{4}\right) \times (LI)$$

and the minimum projection area is

$$\pi \times \left(\frac{I}{2}\right) \times \left(\frac{S}{2}\right)$$

because I and S axes are apparent in the ellipsoid.

The general formula of the volume of the ellipsoid is

$$\left(\frac{4}{3}\right) \times (\pi) \times (abc)$$

where a, b, and c are L, I, and S axes respectively.

The volume of the particle is

$$\frac{\pi}{6} \times (LIS)$$

the volume of the particle equals the volume of the equivalent sphere therefore the volume of the equivalent sphere will also be

$$\frac{\pi}{6} \times (LIS)$$

where $LIS = d^3$ therefore, $d = \sqrt[3]{LIS}$.

The maximum projection area of this sphere is

$$\frac{\pi}{4} \times (\sqrt[3]{LIS})^2$$

therefore equation (6) will be as

$$\text{Maximum projection sphericity} = \frac{\frac{\pi}{4}(\sqrt[3]{LIS})^2}{\frac{\pi}{4}(LI)}$$

which reduces to

$$\text{Maximum projection sphericity} = \sqrt[3]{\frac{L^2 I^2 S^2}{L^3 I^3}} = \sqrt[3]{\frac{S^2}{LI}} \qquad (7)$$

Folk (1946) stated that it is not necessary to compute the value of the maximum projection sphericity. Given the L, I, and S measurements, one can refer to a triangular graph (figure 10) and read the sphericity directly by plotting two diameter ratios $\left(\frac{S}{L}\right)$ and $\left(\frac{L-I}{L-S}\right)$ and interpolating the position of the plotted point between the curving isosphericity contours (Sneed and Folk, 1958).

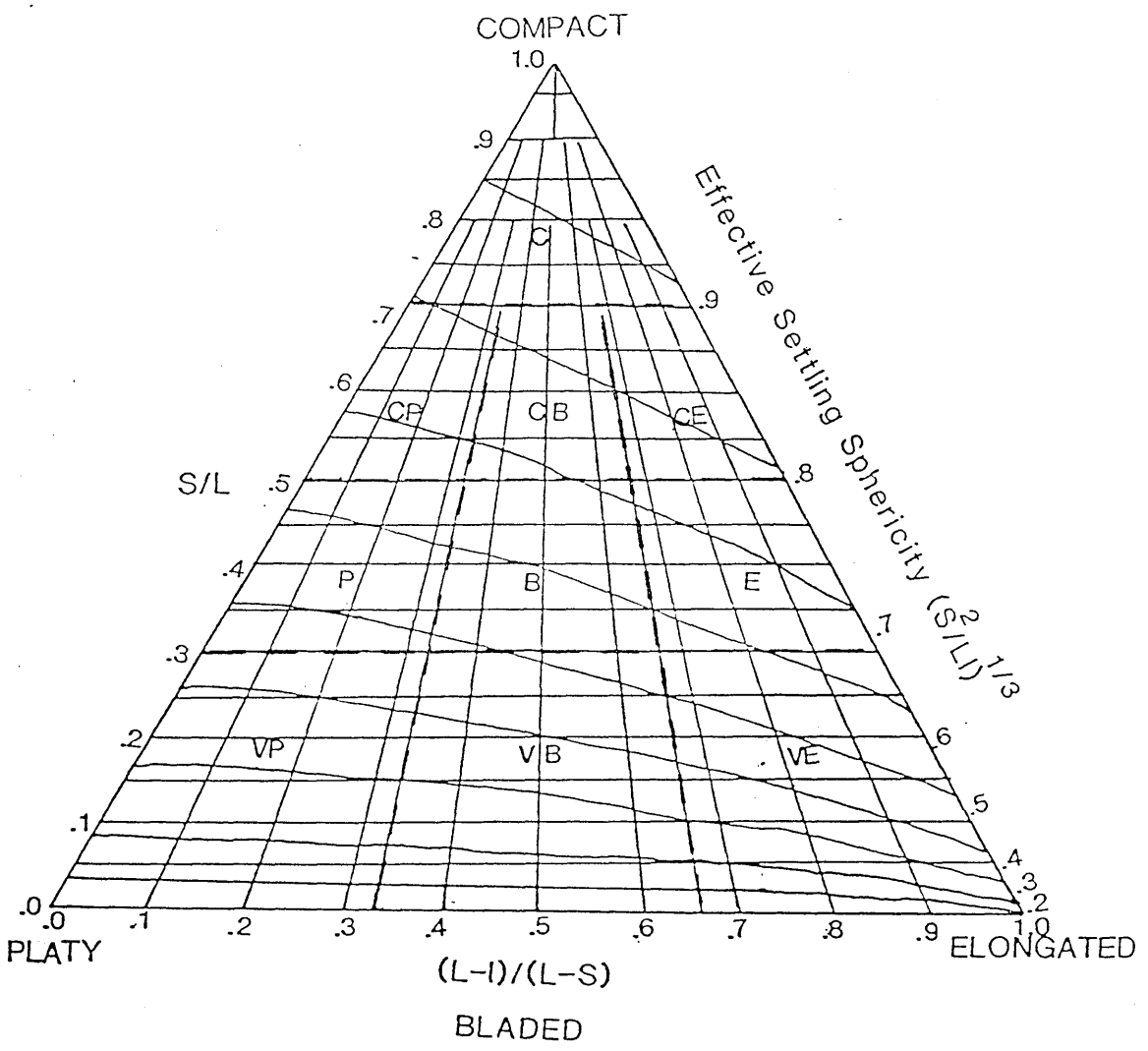
Figure 10. Sphericity-form diagram for particle shapes.

L = Long diameter

I = Intermediate diameter

S = Short diameter.

(Folk, 1980).


COMPACT
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
CI
CP
CB
CE
P
B
E
VP
VB
VE
S/L
Effective Settling Sphericity $(S^2/LI)^{1/3}$
.9
.8
.7
.6
.5
.4
.3
.2
1.0
.0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
PLATY
ELONGATED
(L-I)/(L-S)
BLADED

2.1 Effect of grain shape on permeability

Any change in spherical shape effects permeability by varying the size and shape of the interstices which also causes irregularities in packing (Fraser, 1935 pp.962). At equal diameters of grains and equal porosity, the coefficient of permeability decreases with increasing uniformity of the pore spaces (Furnas, 1929 pp.53). Hence, as the form of the grains departs from that of a true sphere, the permeability will increase (Fraser, 1935). Under turbulent conditions, maximum flow is obtained through a rectangular opening, while minimum flow is obtained through an orifice which approaches an equilateral triangle in form. Other shapes permit a rate of flow intermediate in value between these two extremes. With laminar flow, the relationship is not so simple or so direct, but the greatest flow is also through a rectangular opening (Fraser, 1935, pp.962-963).

Some of the discrepancies between the computed and experimental value of the coefficient of permeability have been assigned to the effect of variations in the rounding of grains. The permeability of even the most angular sands, solely because of their angularity, would probably not be greater than two or three times that of a well-rounded sand (Fraser, 1935).

2.2 Effect of grain shape on porosity-Fraser's (1935) experiment

It is difficult to determine the effect of grain shape on porosity, because of the difficulty in obtaining angular particles of the same size. The porosity of a soil is defined as the ratio of volume of voids to total volume of sample. Fraser (1935) conducted a classic experiment to find the effect of grain shape on porosity which I shall describe in detail because it is so important. I have divided my account of Fraser's experiment into methods, results, and summary.

2.3 Methods

The effect of angularity on porosity was determined under conditions in which other factors were constant, on a series of carefully sized materials which ranged in shape from spheres to flat plates. The materials selected by Fraser were:

1) Lead shot.
2) Sulphur shot.
3) Standard sand (marine).
4) Beach sand.
5) Dune sand.
6) Crushed calcite.
7) Crushed quartz.
8) Crushed halite.
9) Crushed mica.

Lead shot and sulphur shot were used as spherical grains because perfectly spherical sand grains were not obtainable. However Fraser states that the porosities of these two materials at loosest packing are not reliably comparable with those for the other materials, because they differ markedly from the other materials in specific gravity, which affects the compactness of the unjarred assemblages.

The sand was carefully screened and the portion which passed an 18-mesh screen was retained on a 35-mesh screen. This gave a sample with an average diameter of 1.5 mm*. The lead shot and the sulphur shot had this same diameter (1.5 mm).

The porosity was measured before and after the material was compacted by jarring, and under two experimental conditions:

1) When the material was dry.

2) When the material was saturated with water.

1) Dry material.

The porosity was measured under two conditions:

a) Loose dry packing.
b) Compacted dry packing.

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* According to the US standard in use today 18-mesh equals 1.0 mm and 35-mesh equals 0.5 mm (Folk, 1980, p.23 and Twenhofel and Tayler, 1941, p.48), which would not give an average diameter of 1.5 mm. We must assume that either Fraser was using a different standard or there is a printing error in his paper.

a) Loose dry packing.

The material was slowly poured into a weighed and calibrated measuring flask, and allowed to pack naturally until the flask was full. The material was then weighed. The volume of the grains was determined in a pycnometer (an instrument for determining specific gravity), using tetralin (1,2,3,4-Tetrahydro-naphthalene. ($C_{10}H_{12}$)) as the immersion fluid (figure 11). The porosity was then easily computed

$$\text{porosity} = \frac{\text{volume of void}}{\text{total volume of sample}}.$$

b) Compacted dry packing.

After the porosity was found by loose dry packing, the material was compacted by tapping the flask, more material being added as needed, and the jarring continued until settling ceased. The flask was again weighed and the porosity found.

2) Material saturated with water.

The porosity was again computed under two conditions (as for dry packing).

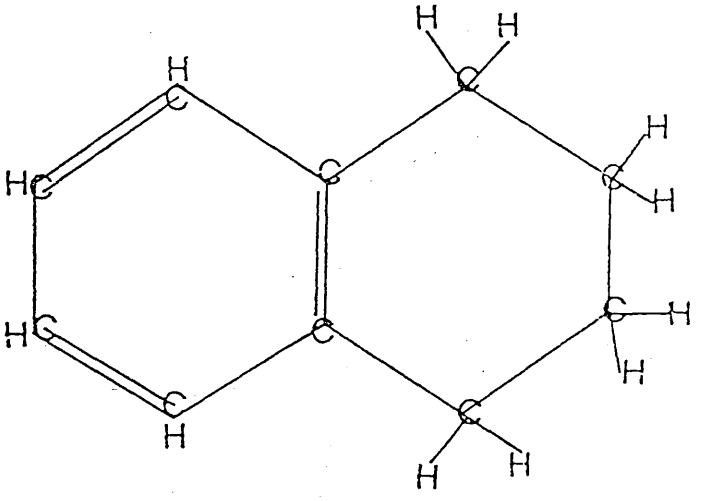
a) Loose wet packing.

The material was first immersed in water to wet it thoroughly and remove adhering air. A measuring flask was filled with water and the material slowly poured in and allowed to settle naturally. The material was then dried at 110°C, allowed to cool and take up moisture from the air and weighed. The volume was determined and the porosity was computed.

b) compacted wet packing.

The material was compacted by jarring (as in paragraph 1b) the volume was determined and the porosity computed.

Figure 11. Formula of Tetralin (C_{10} H_{12}), 1,2,3,4-Tetrahydro-naphthalene (C_{10} H_{12}).



2.2.2 Results

The results of Fraser's experiment, (table 4), show the effects of grain shape on porosity, to be as follows.

1) The porosity of wet packing is higher than the porosity of dry packing.

2) The porosity of beach sand does not differ widely from that of spherical grains.

3) Well-rounded sands show little variation in porosity because the deviation in their rounding is limited.

4) Flat and needle like forms have the greatest effect on porosity (crushed mica). In these grains, the weight of the overlying material produces less compression in underlying areas than in materials of a spherical shape, which, for a given volume, are packed more tightly and are therefore heavier.

5) Porosity usually increases with angularity, except when the grains are mildly and uniformly disc shaped, i.e. when it decreases slightly. Under wet loose packing, beach sand and dune sand have porosities of 46.55% and 44.93% respectively. Under dry loose packing crushed calcite and crushed quartz have porosities of 50.50% and 48.13% respectively.

6) Aggregates of flat particles offer least resistance to compaction by external pressures. This may explain in part, the constant relationship observed in clays between porosity and external pressures, as represented by the weight of overlying rock.

Material	specific gravity	porosity (%)			
		dry packing		wet packing	
		loose	compacted	loose	compacted
lead shot	11.21	40.06	37.18	42.40	38.89
sulphur shot	2.024	43.38	37.35	44.14	38.24
standard sand (marine)	2.681	38.52	34.78	42.96	35.04
beach sand	2.658	41.17	36.55	46.55	38.46
dune sand	2.681	41.17	37.60	44.93	39.34
crushed calcite	2.665	50.50	40.76	54.50	42.74
crushed quartz	2.650	48.13	41.20	53.88	43.96
crushed halite	2.180	52.05	43.51	.....	.....
crushed mica	2.837	93.53	86.62	92.38	87.28

Table 4. Influence of grain shape on porosity. (Fraser 1935, p.936).

Note: porosity, being a ratio, has no dimensions.

7) Loosely packed material (dry or wet) has a higher porosity than compacted material.

2.2.3 Summary

Fraser's experiment tested the effect of grain shape on porosity using nine different materials having different shapes. The experiment was run under two conditions:

1) When the material was dry.
2) When the material was wet.

He also tested the effect of loose packing and compact packing on porosity. The material used had average diameter of 1.5 mm. Lead shot and sulphur shot were used as a basis because perfectly spherical sand grains were not obtainable. However these two materials have higher specific gravities than other materials and this affected compaction. Fraser concluded the following:

1) wet material has higher porosity than dry material.
2) Loosely packed material (dry or wet) has a higher porosity than compacted material.
3) In general, porosity increases with increasing particle angularity.

III) PACKING

Packing has to do with the manner of arrangement or spacing of the solid particles in a sediments (Pettijohn, 1976).

The geometrical arrangement with the greatest volume of unit void would be the arrangement with the highest porosity. Likewise, the arrangement with the smallest volume of unit void would have the lowest porosity. The loosest manner of systematic arrangement of uniform solid spheres in a clastic sediment is characterised by a unit cell that is a cube whose eight corners are the centres of the spheres involved. This arrangement of spheres is called the cubic packing or open packing. An aggregate with cubic packing has the maximum porosity (47.64%). The tightest manner of systematic arrangement of uniform solid spheres in a clastic sediment is characterised by a unit cell of six planes passed through eight sphere centres situated at the corners of a regular rhombohedron. This arrangement of spheres is called rhombohedral packing. An aggregate with rhombohedral packing has the minimum porosity (25.95%) which can be produced without distortion of the grains (Graton and Fraser 1935; Kahn, 1956; Pettijohn, et al. 1984; Bates and Jackson, 1987).

In a system of closely packed spheres of uniform size the unit arrangements of spheres are of two kinds. The first consists of four spheres, the centres of which from a tetrahedron (fig. 12a). The second consists of six spheres, four of which are equatorial with their centres at the corners of a square while the remaining two rest above and beneath and thus may be considered as polar (fig. 12b), the centres of the spheres in this unit forming a double pyramid with a square base and all of its sides of length twice the radius of a sphere. These two unit arrangements share common spheres so that the

Figure 12. The unit arrangements of spheres basic to a system of closely packed spheres of uniform size. a, four spheres enclosing a curvilinear tetrahedron of void; b, six spheres enclosing a curvilinear cube of void; c, a system of seven spheres combining both the tetrad and six sphere units (Webb, 1958).

a
b
c

addition of one sphere touching any three of the unit of six gives a system of seven spheres which is a combination of both (fig. 12c). By the addition of their spheres these units are repeated and will form a close-packed system of infinite extent.

The basic arrangement of uniform spheres in a close packed system is three spheres in contact and these enclose a curvilinear triangular pore between their points of contact. All interstitial spaces are entered through these triangular pores and in one of these pores will fit a sphere of radius 0.164a where a is the radius of the large spheres. The porosity of the system is the proportion of the total volume that is interstitial space, but the rate of flow through the system of a liquid of given fluidity depends not on porosity but on the area of cross-section of triangular pores. In a close packed system of mixed spheres, therefore, the presence of spheres of radius 0.414a and 0.225a in the appropriate interstitial spaces will principally cause a reduction in porosity or void ratio rather than rate of flow. On the other hand, spheres of 0.164a lodged in the triangular apertures will greatly reduce the rate of flow of a liquid through the system, but have little effect on porosity (Webb, 1958; Webb, 1969).

In general, the tighter the packing density the lower its effective porosity and hence the lower its permeability (Fraser, 1935; Graton and fraser, 1935; Kahn, 1956; Webb, 1958; Webb, 1969; Pettijohn, 1975; HSU, 1977; Collinson and Thompson, 1982; Pettijohn et al. 1984).

IV) Void ratio

Soil is made up of various-sized particles packed together, the spaces between particles being known as voids (figure 13, Smith 1981). The voids generally contain a mixture of air and water, but in certain circumstances may contain air alone or water alone. The void ratio (e) is the ratio of the volume of voids (Vv) to the volume of solids (Vs).

$$e = \frac{V_v}{V_s}$$

(Smith, 1981; Capper, et al. 1966).

Taylor (1948), Wallace (1948) and Lambe and Whitman (1979) have stated that the void ratio has an important effect on permeability. When a soil is compressed or vibrated, the volume occupied by its solid constituents remains practically unchanged, but the volume of the voids decreases. As a consequence, the permeability of the soil also decreases. The influence of the void ratio on the permeability is illustrated by figure 14 (Terzaghi and Peck, 1967).

V) Composition

Soil composition is of limited importance in the permeability of some soil types such as silts, sands and gravels (mica and organic matter are two exceptions). It is of major importance in clays (Lambe and Whitman, 1979; Das 1985).

Over the range of water contents, the permeability of all the clay minerals is less than about 1×10^{-5} cm/sec and may range to values less than 1×10^{-10} cm/sec for some of the monovalent ionic forms of the smectite minerals. The smectite is a name for the montmorillonite group of clay minerals. The usual range observed for natural clay soils is 1×10^{-6} to 1×10^{-8} cm/sec. For clay mineral compared at the

Figure 13. Soil sample showing voids between solids (Smith 1981).

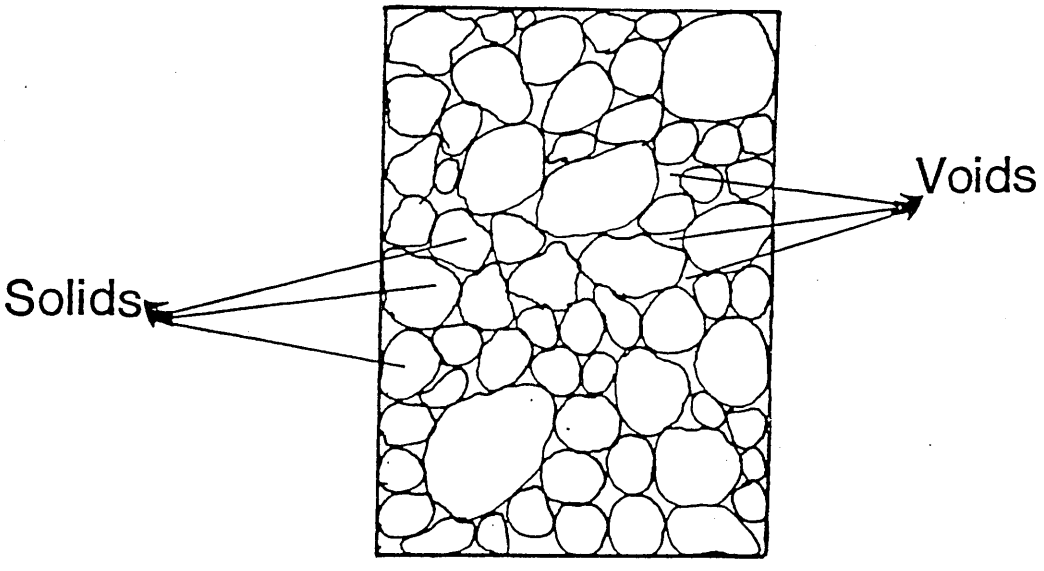
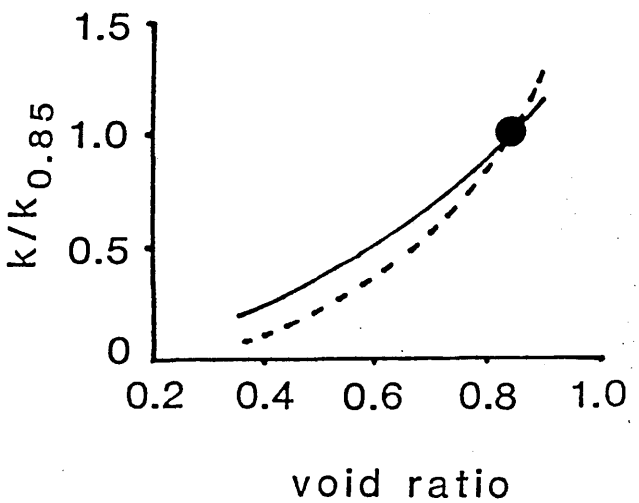

Voids
Solids

Figure 14. Relationship between void ratio and permeability of mixed-grained sand (full line) and soil with flakey constituents (dash line), where the abscissa represents the void ratio. The ordinate represents the ratio $k/k_{0.85}$ between the coefficient of permeability k of the soil at any given void ratio e and that of the same soil at a void ratio of 0.85 (Terzaghi and Peck 1967).


1.5
1.0
0.5
0
$k/k_{0.85}$
0.2
0.4
0.6
0.8
1.0
void ratio

same water content, the permeability coefficients are in the order smectite (montmorillonite) < attapulgite < illite < Kaolinite (Mitchell, 1976).

Figure 15* shows the extent to which composition affects clay permeability. The data in the figure indicate that of the common exchangeable ions, sodium is the one that gives the lowest permeability to a clay. It also shows that at a void ratio as high as 15, sodium montmorillonite has a permeability less than 10^{-7} cm/sec. Sodium montmorillonite, being one of the least permeable soil minerals, is therefore widely used by engineers as an additive to other soils to reduce their permeability. The effect of exchangeable ions on permeability decreases with a decrease in the ion exchange capacity of a soil (Mitchell, 1976; Lambe and Whitman 1979).

(VI) Fabric

Fabric may be defined as the arrangement in space of the components of a soil, the various particles and the voids between them (Lee, et al.,1983).

It is important to note that fabric is closely related to packing. Fabric is the way in which the grains are put together to make an aggregate, and it can be viewed as a natural structure of a soil, whereas packing (see page 123) has to do with the manner of arrangement or spacing of the solid particles in a sediment (Pettijohn, 1976; Pettijohn, et al. 1984).

The fabric component of structure is one of the most important sediment properties influencing permeability, especially in

---

* This figure is taken from Lambe and Whitman (1979) as Cornell University (1951), the original reference, appears to be impossible to obtain.

Figure 15. The effect of soil composition on permeability.

✶ Sodium montmorillonite and Kaolinite

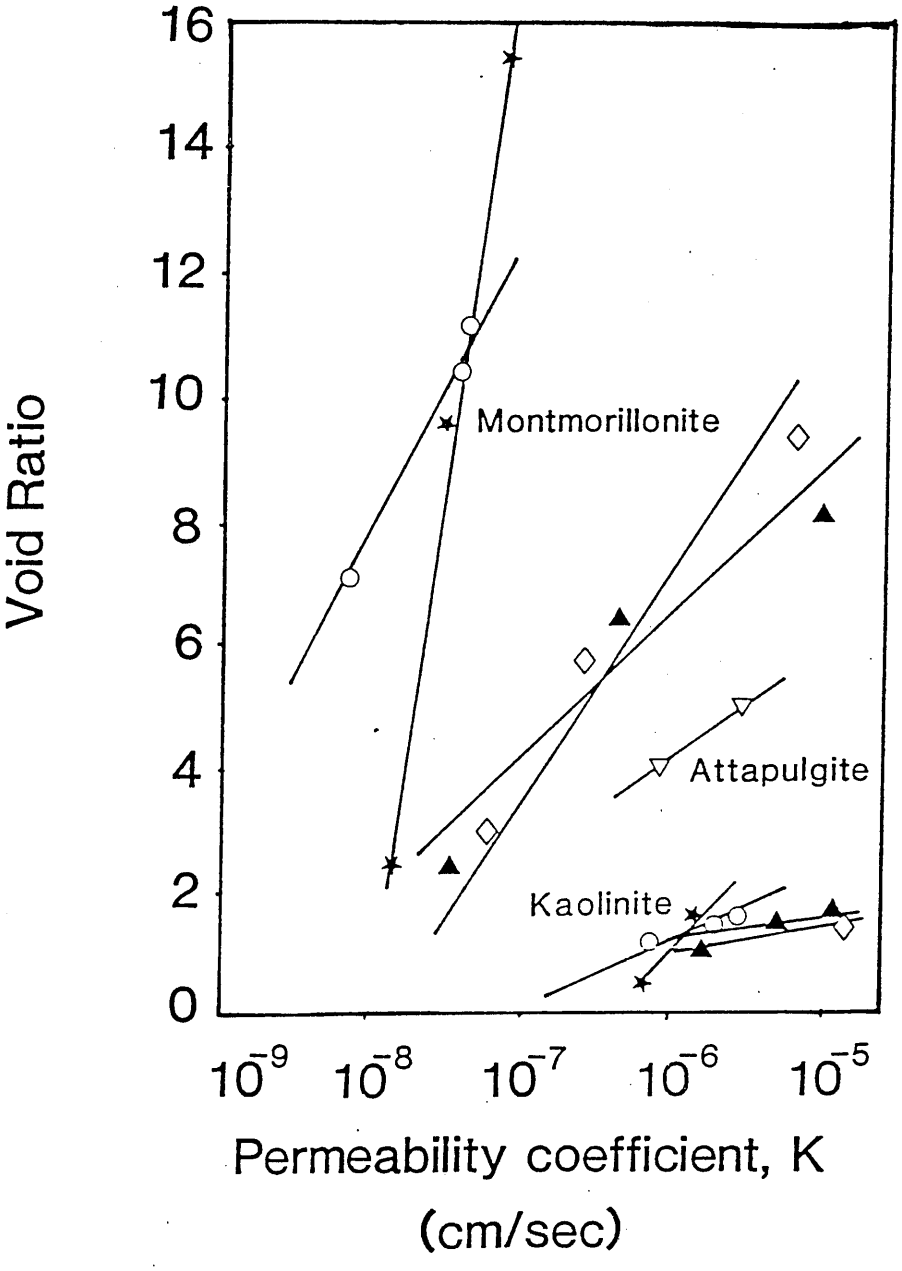
○ Potasium montmorillonite Kaolinite

▲ Calcium montmorillonite and Kaolinite

□ Hydrogen montmorillonite and Kaolinite

▽ Natural montmorillonite and Kaolinite

(Lambe and Whitman, 1979).


16
14
12
10
8
6
4
2
0
Void Ratio
Montmorillonite
Attapulgite
Kaolinite
10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5}
Permeability coefficient, K
(cm/sec)

fine-grained soils (Lambe and Whitman, 1979). Comparing soil specimens at the same void ratio, we find that the specimens which are in the most flocculated state will have the highest permeability, and the ones in the most dispersed state will have the minimum permeability (Lambe 1955; Lambe and Whitman 1979).

The main factor affecting permeability, however, is that in a flocculated soil there are some large channels available for flow. Since flow through one large channel will be much greater than flow through a number of small channels having the same total channel area, it is readily apparent that the larger a channel for a given void volume, the higher the permeability (Lambe and Whitman, 1979).

VII) Degree of saturation

The ratio of volume of water (V_w) to volume of voids (V_v) is known as the degree of saturation.

$$\text{Degree of saturation } S_r = \frac{V_w}{V_v}$$

The degree of saturation S_r, expressed in percent, and falls in the range of $0 < S_r < 100$ (Cernica, 1982).

$$\text{Percentage saturation} = S_r \times 100$$

Degree of saturation should not be confused with water content. In the end of this section I am going to define the water content and to explore the relationship between water content, degree of saturation and void ratio.

Below the water table the soil voids are usually completely filled with water, in which case the degree of saturation is 1, or the percentage saturation 100 per cent (Smith, 1981). The degree of saturation of a soil has an important influence on its permeability.

The higher the degree of saturation, the higher the permeability (Wallace, 1948). Wallace (1948) has reported a most interesting experiment relating degree of saturation to permeability. Five samples were used in the experiment. These were two different samples of Union Falls sand, called "A" and "B", a coarse Ottawa sand, a medium Fort Peck sand and a very fine Franklin Falls sand.

The sample was dried and allowed to cool to room temperature in a dissicator. After mixing the sample thoroughly in its evaporating dish to ensure a uniform distribution and to break up all lumps formed in drying, the sample was placed in a lucite tube in layers 0.7 cm high. Each increment in the first 5 cm of sample was given twelve blows of 5 cm free drop with a 190 gram tamper which closely fitted the sides of the tube. The increments in the remaining length of the sample, except for the top 5 cm, were given fifteen blows. The increments in the top 5 cm of the sample were given seventeen blows (Lucite tube = permeameter tube). After the sample was placed in the tube, the tube and the sample were weighed to 0.1 gram. This was to check for loss of sample during the test.

All samples were saturated from the bottom upward by capillary action with distilled water. The distilled water had been agitated with air and left in contact with air for 24 hours before the experiment, so that it was 100% saturated with air. Great care was taken to keep the free water surface below the level of the capillary fringe.

After several determinations of the rate of flow through the sample of the 100% air saturated distilled water, de-aired water was passed into the apparatus through the capillary tube. The flow through the capillary tube was regulated so that there was a constant flow of

water through the constant head tube.

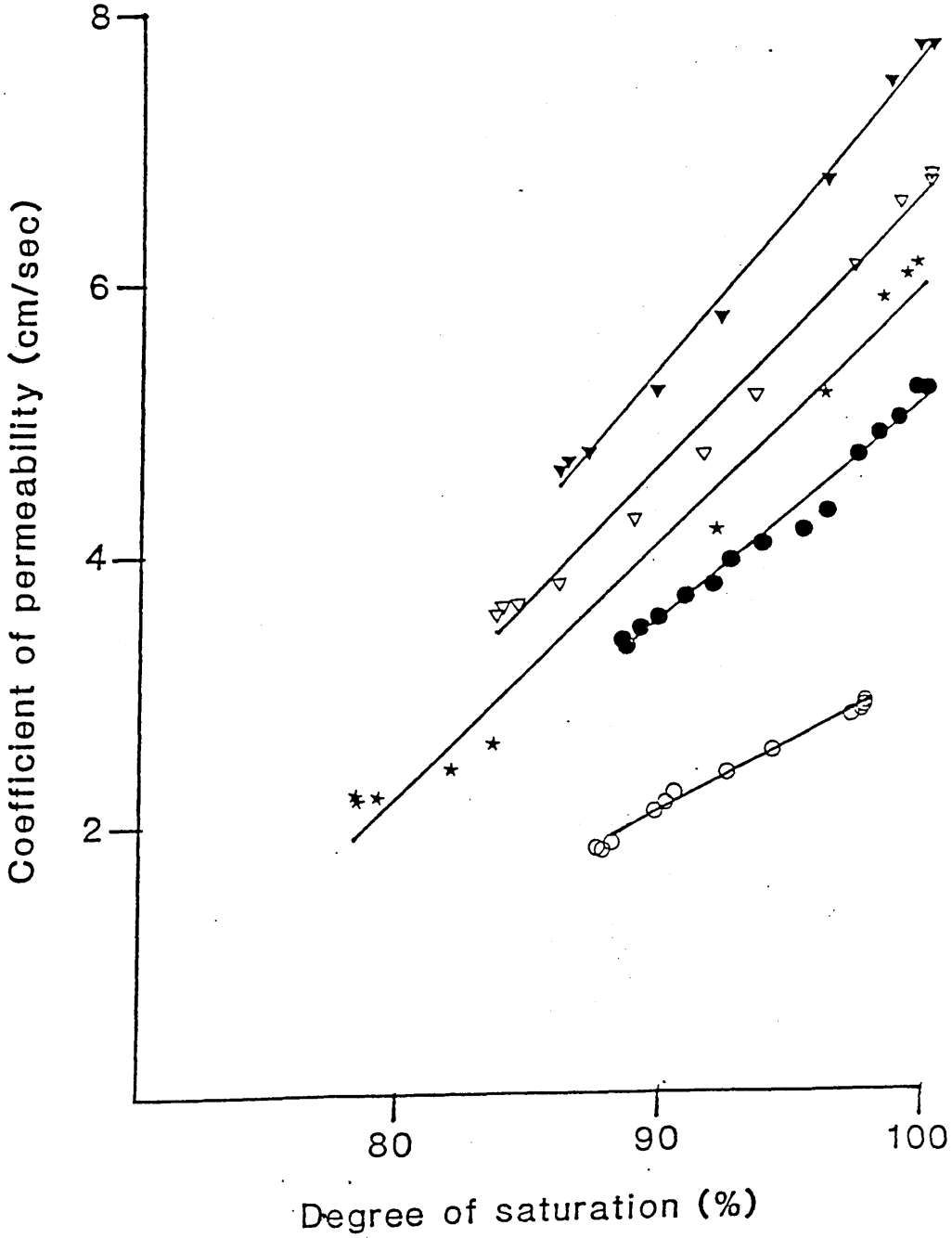
The rate of flow was measured at several intervals using a 100 ml burette and a timer. If the flow was very slow, a 25.0 ml burette was used.

The time was recorded when the water level in the burette was at 90, 70, 50, 30, 10 and 0 ml, and the average rate of flow was calculated from these.

After each run the permeameter tube was weighed. These weights progressively increased. Wallace assumed that the increase in weight was caused by the voids becoming progressively more saturated with time, until when the weight became constant, the sediment was fully saturated. This proved to be correct. The weights, with the initial weight, were used to calculate the degree of saturation at each time interval. When the rate of flow through the sample and the weight of the apparatus became constant, the test was stopped, the apparatus disassembled, and the permeameter tube and wet sample weighed, to the nearest 0.1 gram, after carefully drying the exposed surfaces with absorbent paper tissue. The sample was then dried and weighed as before. The result of these experiments showed that the permeability of all the soils tested increased with increasing degree of saturation and void ratio (figure, 16). The data in this figure are taken from Wallace (1948) (appendix 2). I have analysed the data by regression analysis. The resultant regression lines are plotted in figure 16 and their statistical analysis is given in table 5. The correlation coefficients of the five stations are all highly significant and the t.test probabilities are all significant. This means that degree of saturation has a great effect on permeability for all five sand samples. A detailed listing of the minitab print out

Figure 16. Relationship between permeability coefficient (cm/sec) and degree of saturation (%) of sediments. Original data taken from Wallace (1948). Regression lines were fitted by E. Hilal to this data using the Minitab statistical package. Details of equations and statistical analyses are given in table 5. Each sand has a different void ratio, and a different scale factor to be read given the y axis:

Sand		Void ratio	Scale factor
Union Falls A	▼	0.65	10^{-3}
Union Falls B	▽	0.66	10^{-3}
Fort peck sand	✦	0.58	10^{-3}
Franklin Falls	●	0.73	10^{-4}
Ottawa sand	○	0.48	10^{-2}


8
6
4
2
Coefficient of permeability (cm/sec)
80
90
100
Degree of saturation (%)

Station	Regression equation	Regression coefficient
Union Falls A	y = 0.2328x - 15.5078	0.2328
Union Falls B	y = 0.2011x - 13.4054	0.2011
Fort peck	y = 0.1904x - 12.9989	0.1904
Franklin Falls	y = 0.1621x - 11.0451	0.1621
Ottawa	y = 0.1028x - 7.1447	0.1028

Station	Correlation coefficient	t.test	Degree of freedom	Probability	Significant
Union Falls A	0.997	32.87	8	P<0.001	****
Union Falls B	0.994	27.31	10	P<0.001	****
Fort peck	0.985	17.32	10	P<0.001	****
Franklin Falls	0.991	27.70	15	P<0.001	****
Ottawa	0.996	33.90	11	P<0.001	****

Table 5. The statistical analysis of Wallace's (1948) data.

that I obtained for the regression analyses is given in appendix 3.

VIII) Biological Effects

Permeability is one of the important parameters to be measured in bioturbation studies. Permeability is affected by burrowing invertebrates whose burrows may increase sediment permeability (Smith et al. 1944; Nowell et al. 1981; Weaver and Schulteiss, 1983; Meadows and Tufail 1986; Meadows and Tait 1989). Permeability also affects the distribution of intertidal burrowing invertebrates (Holme, 1949; Webb, 1958; 1969; Ruello, 1973) since they are dependent on the water held between sediment particles during low tide.

Permeability is also affected by micro-organisms (Plummer et al., 1944; Alison, 1947; McCalla, 1950; Webb, 1969; Jenneman et. al., 1984; Shaw et. al., 1985; Meadows and Tufail, 1986) that produce mucopolysaccharides which fill sediment voids and thereby reduce permeability.

Meadows and Tufail (1986), report a laboratory experiment showing the effect of the growth of micro-organisms on sediment permeability. The experiment was conducted on cores of sediment containing enrichment media under controlled light and dark conditions to stimulate the growth of micro-organisms. The results of this experiment showed that the permeability of the cores containing the enrichment culture became progressively reduced as the experiment proceeded. This difference is caused by the growth of micro-organisms in the interstices of the sediment, while control cores containing no enrichment media showed only a slight reduction in permeability.

THE RELATIONSHIP BETWEEN WATER CONTENT, DEGREE OF SATURATION AND VOID RATIO

The following section describes the relationship between water content, degree of saturation and void ratio. The water content (w) of a soil sample is measured by calculating the ratio of the weight of water in the sample to the weight of solids (equation 7 below). The sample is weighed, dried at a temperature of 105° to 110° C, and weighed again. The loss in weight represents the weight of water in the sample. The water content is a dimensionless ratio. It is more often quoted as a percentage (equation 8 below) and in practice ranges between 0% for a completely dry terrestrial soil and 120% for a marine silty clay (Table 6) (Papadakis, 1941; Baver, 1956; Baver et al. 1972; Scott,1974; Capper and Cassie, 1976; Atkinson and Bransby, 1978; Lambe and Whitman, 1979; Dunn, et al.,1980; Smith, 1981; Cernica, 1982; and Lee, et al., 1983).

$$w = \frac{\text{weight of water } (W_w)}{\text{weight of dry soil } (W_s)} \qquad (7)$$

$$\text{Percentage water content} = w \times 100 \qquad (8)$$

In sand the percentage water content varies between 10 and 30%, while in clay it ranges from 10 to over 300% (Cernica, 1982).

The degree of saturation (Sr) is defined on page 135 and is calculated using the following equation:

$$Sr = \frac{\text{volume of water } (V_w)}{\text{volume of voids } (V_v)} \qquad (9)$$

The ratio of volume of voids (V_v) to volume of solids (V_s) is known as the void ratio (e) (equation 10):

$$e = \frac{V_v}{V_s} \qquad (10)$$

soil type	water content w(%)	void ratio e(%)	porosity n(%)	Bulk density (t/m)	dry density
Dense, saturated, angular, well graded quartz sand.	8	20	17	2.4	2.2
Loose, saturated, angular well graded quartz sand.	34	20	47	1.9	1.4
Dense, saturated, rotund, uniformly graded quartz sand.	16	40	30	2.2	1.9
Loose, saturated, rotund, uniformly graded quartz sand.	38	100	50	1.8	1.3
Normally consolidated marine silty clay.	120	325	76	1.4	0.6
Lightly overconsolidated clay.	40	100	50	1.8	1.3
Heavily overconsolidated (London) clay.	23	62	39	2.1	1.7

Table 6. Interrelationship between void ratio, porosity, bulk density, and dry density, and water content of a saturated soil (Lee, et al. 1983, p.12).

The void ratio is expressed as a dimensionless number or as a percentage (Lee, et al., 1983) and ranges between 20% for a dense, saturated, angular, well graded quartz sand and 325% for a normally consolidated marine silty clay (Table 6). Table 6 also gives porosity, bulk density, and dry density for comparison (Cernica, 1982; Lee, et al., 1983, p. 12).

It is thus clear, as seen from equations 7 and 9, that the water content and degree of saturation are not equal but are related to each other.

Both the degree of saturation and the water content of a soil may have a significant effect on other characteristics and behaviour of the soil. This is particularly true of a fine grained soil. For example, a high water content may greatly reduce the shear strength of a clay stratum and/or its bearing capacity. On the other hand the degree of saturation may appreciably influence the amount and rate of consolidation (Cernica, 1982).

In order to show that a relationship exists between water content, degree of saturation and void ratio I have considered three cases A, B and C. A, B and C each contain three different situations, A1, A2, A3, B1, B2, B3 and C1, C2 and C3 (figure 17a, 17b, 17c and table 7). B1 is taken from Smith (1981, p.6). I have derived the remaining 8 myself.

Cases A, B and C have the same volume of sample (0.0183 m^3) and specific gravity of solids (2.65) but they differ in the following two ways:

1) weight of dry soil.

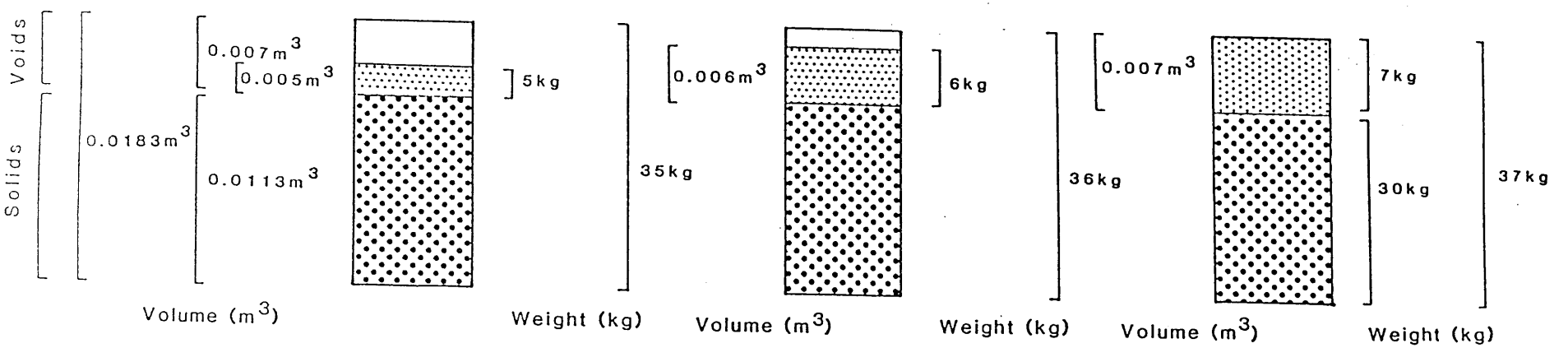
2) volume of voids.

A1, A2 and A3 have the same void ratio (0.6195). They differ from

Figure 17a. Case A consist of three examples A1, A2 and A3. These examples have same void ratio (0.6195). They differ from each other in the following ways:

1) Their percentage water contents are 16.7, 20 and 23.3 respectively.

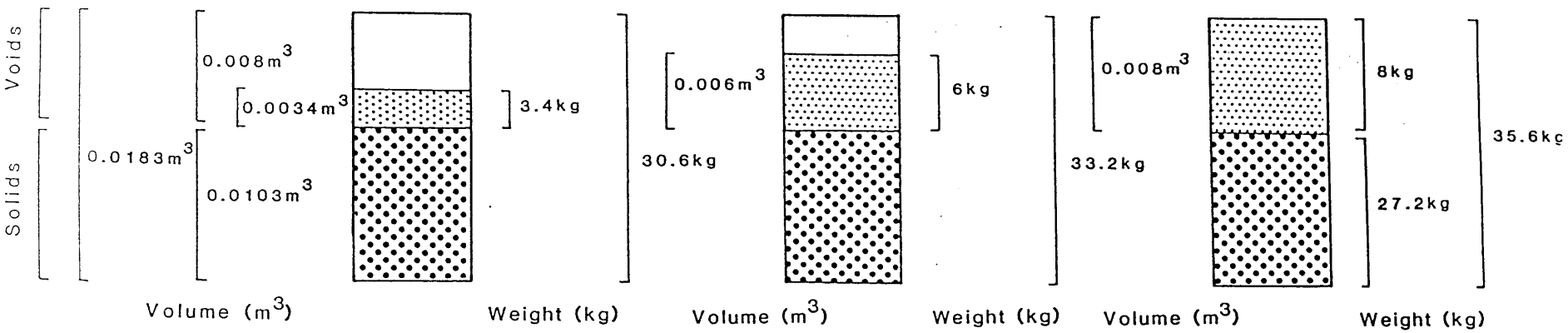
2) Their percentage saturations are 71.4, 85.7 and 100 respectively.



Water Content	16.7%	20%	23.3%
Degree of Saturation	71.4%	85.7%	100%
Void ratio	0.6195	0.6195	0.6195

Figure 17b. Case B consist of three examples B1, B2 and B3. These examples have same void ratio (0.7767). They differ from each other in the following ways:

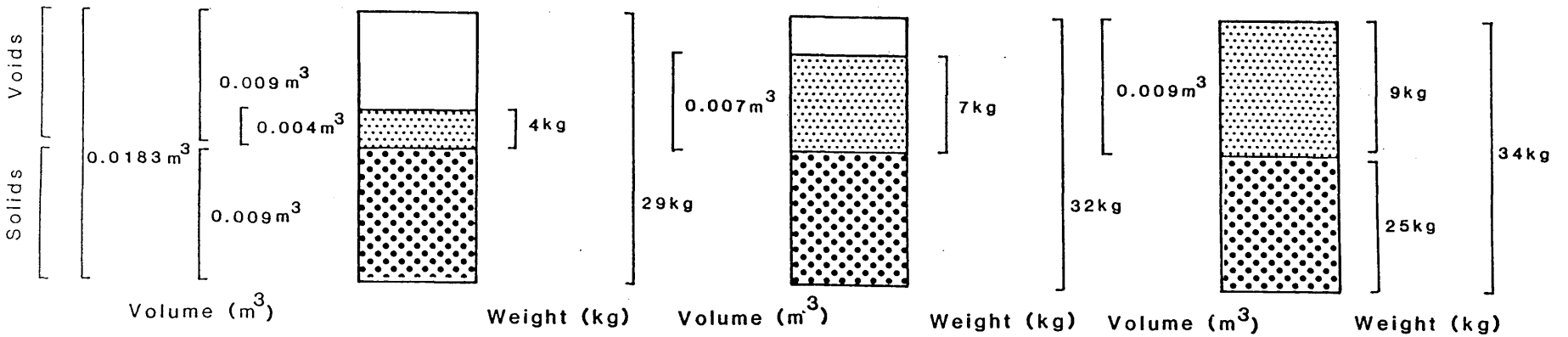
1) Their percentage water contents are 12.5, 22.1 and 29.4 respectively.
2) Their percentage saturations are 42.5, 75 and 100 respectively.



Water Content	12.5%	22.1%	29.4%
Degree of Saturation	42.5%	75%	100%
Void ratio	0.7767	0.7767	0.7767

Figure 17c. Case C consist of three examples C1, C2 and C3. These examples have same void ratio (1). They differ from each other in the following ways:

1) Their percentage water contents are 16, 28 and 36 respectively.
2) Their percentage saturations are 44.4, 77.8 and 100 respectively.



Water Content	16%	28%	36%
Degree of Saturation	44.4%	77.8%	100%
Void ratio	1	1	1

cases	examples	void ratio	degree of saturation	water content
A	1	0.6195	100%	23.3%
	2	0.6195	85.7%	20%
	3	0.6195	71.4%	16.7%
B	1	0.7767	100%	29.4%
	2	0.7767	75%	22.1%
	3	0.7767	42.5%	12.5%
C	1	1	100%	36%
	2	1	77.8%	28%
	3	1	44.4	16%

Table 7. The relationship between void ratio, degree of saturation and water content.

each other in the following ways:

1) Their percentage water contents are 16.7, 20 and 23.3 respectively.

2) Their percentage saturations are 71.4, 85.7 and 100 respectively.

B1, B2 and B3 have the same void ratio (0.7767). They differ from each other in the following ways:

1) Their percentage water contents are 12.5, 22.1 and 29.4 respectively.

2) Their percentage saturations are 42.5, 75 and 100 respectively.

C1, C2 and C3 have the same void ratio (1). They differ from each other in the following ways:

1) Their percentage water contents are 16, 28 and 36 respectively.

2) Their percentage saturations are 44.4, 77.8 and 100 respectively.

The following analysis shows how these figures were calculated. In this analysis the density of pure water is taken to be 1.0. In the analysis it is also necessary to convert from kg water to m^3 water. This conversion is derived as follows.

$$1 \text{ kg} = 1000 \text{ gm} \qquad (11)$$

and

$$1 \text{ gm} = 1 \text{ cm}^3 = 1 \text{ ml} \qquad (12)$$

also

$$1 \text{ L} = 1000 \text{ ml} \qquad (13)$$

and

$$1 \text{ m}^3 = 10^6 \text{ cm}^3 \qquad (14)$$

Therefore,

$$10^6 \text{ cm}^3 = 10^6 \text{ ml} \qquad (15)$$

From equations 13, 14 and 15

$$10^6 \text{ ml} = 10^3 \text{ L}$$

and

$$10^6 \text{ ml} = 1000 \text{ L} = 1 \text{ m}^3.$$

Hence to convert from litre to m^3 we have to divide by 1000.

CASE A

Example A1

A sample weighing 35 kg had a volume of 0.0183 m^3. The weight was reduced to 30 kg after drying in an oven. The specific gravity of the solid was found to be 2.65. The void ratio, degree of saturation and water content w determined as follows:

Specific gravity (G_s) = 2.65

Weight of sample = 35 kg

Volume of sample = 0.0183 m^3

Weight of dry soil (W_s) = 30 kg

Weight of water (W_w) = 35 - 30 = 5 kg = 5 L water = 0.005 m^3 water = volume of water (V_w),

Because the specific gravity of water is taken to be 1.0.

Volume of water (V_w) = weight of water (W_w)

$$\text{Specific gravity of particles}(G_s) = \frac{\text{Weight of solid particles } (W_s)}{\text{Weight of an equal volume of water}}$$

where weight of an equal volume of water equals weight of water displaced by solids. Therefore,

$$G_s = \frac{W_s}{V_s \gamma_w}$$

and

$$\text{Volume of solids } (V_s) = \frac{W_s}{G_s \gamma_w} = \frac{30}{2.65 \times 1000} = 0.0113 \text{ m}^3$$

where

γ_w = density of water.

Volume of voids (V_v) = 0.0183 - 0.0113 = 0.007 m^3

$$\text{Void ratio} = \frac{V_v}{V_s} = \frac{0.007}{0.0113} = 0.6195$$

where

V_s = volume of sands.

$$\text{Percentage degree of saturation} = \frac{V_w}{V_v} = \frac{0.005}{0.007} \times 100 = 71.4\%$$

$$\text{Percentage water content} = \frac{W_w}{W_s} = \frac{5}{30} \times 100 = 16.7\%$$

Example A2

The sample used for example A2 had the same weight of dry soil and void ratio (30 kg and 0.6195 respectively) as A1 but a different weight (36 kg). The water content and the degree of saturation are obtained as in example A1:

Weight of water (W_w) = 36 - 30 = 6 kg = 6 L water = 0.006 m^3 water

= volume of water (V_w),

because the specific gravity of water is taken to be 1.0.

$$\text{Percentage degree of saturation} = \frac{0.006}{0.007} \times 100 = 85.7\%$$

$$\text{Percentage water content} = \frac{6}{30} \times 100 = 20\%$$

Example A3

The sample used for example A3 had the same weight of dry soil and void ratio as A1 and A2 but a different weight (37 kg). The water content and the degree of saturation are determined as in example A1:

Weight of water (W_w) = 37 - 30 = 7 kg = 7 L water = 0.007 m^3 water

= volume of water (V_w),

because the specific gravity of water is taken to be 1.0.

$$\text{Percentage degree of saturation} = \frac{0.007}{0.007} \times 100 = 100\%$$

$$\text{Percentage water content} = \frac{7}{30} \times 100 = 23.3\%$$

CASE B

Example B1

A sample weighing 30.6 kg, had a volume of 0.0183 m^3. After drying in an oven, its weight was reduced to 27.2 kg. The specific gravity of the solid was found to be 2.65. This example is the one given in Smith 1981, p 6). The void ratio, degree of saturation and water content can be determined as in example A1:

Specific gravity (G_s) = 2.65

Weight of sample = 30.6 kg

Volume of sample = 0.0183 m_3

Weight of dry soil (W_s) = 27.2 kg

Weight of water (W_w) = 30.6 - 27.2 = 3.4 kg = 3.4 L water = 0.0034 m^3

water = volume of water (V_w),

because the specific gravity of water is taken to be 1.0.

$$\text{Volume of solids } (V_s) = \frac{27.2}{2.65 \times 1000} = 0.0103 \text{ m}^3$$

$$\text{Volume of voids } (V_v) = 0.0183 - 0.0103 = 0.008 \text{ m}^3$$

$$\text{Void ratio (e)} = \frac{0.008}{0.0103} = 0.7767$$

$$\text{Percentage degree of saturation } (S_r) = \frac{0.0034}{0.008} \times 100 = 42.5\%$$

$$\text{Percentage water content (w)} = \frac{3.4}{27.2} \times 100 = 12.5\%$$

Example B2

The sample used for example B2 had the same weight of dry soil and void ratio (27.2 kg, and 0.7767 respectively) as B1 but a different weight (33.2 kg). The degree of saturation and the water content can be obtained as in example A1:

Weight of water (W_w) = 33.2 - 27.2 = 6 kg = 6 L water = 0.006 m^3 water = volume of water (V_w),

because the specific gravity of water is taken to be 1.0.

$$\text{Percentage degree of saturation} = \frac{0.006}{0.008} \times 100 = 75\%$$

$$\text{Percentage water content} = \frac{6}{27.2} \times 100 = 22.1\%$$

Example B3

The sample used for example B3 had the same weight of dry soil and void ratio as B1 and B2 but a different weight (35.6 kg). The water content and the degree of saturation are calculated as in example A1:

Weight of water (W_w) = 35.2 - 27.2 = 8 kg = 8 L water = 0.008 m^3 water = volume of water (V_w),

because the specific gravity of water is taken to be 1.0.

$$\text{Percentage degree of saturation} = \frac{0.008}{0.008} \times 100 = 100\%$$

$$\text{Percentage water content} = \frac{8}{27.2} \times 100 = 29.4\%$$

CASE C

Example C1

A sample weighing 29 kg, had a volume of 0.0183 m^3. After drying in an oven, its weight was reduced to 25 kg. The specific gravity of the solid was found to be 2.65. The void ratio, degree of saturation and water content can be determined as in example A1:

Specific gravity (G_s) = 2.65

Weight of sample = 29 kg

Volume of sample = 0.018 m^3

Weight of dry soil (W_s) = 25 kg

Weight of water (W_w) = 29 - 25 = 4 kg = 4 L water = 0.004 m^3 water = volume of water (V_w),

because the specific gravity of water is taken to be 1.0.

$$\text{Volume of solids } (V_s) = \frac{25}{2.65 \times 1000} = 0.009 \text{ m}$$

$$\text{Volume of voids } (V_s) = 0.0183 - 0.009 = 0.009 \text{ m}^3$$

$$\text{Void ratio} = \frac{0.009}{0.009} = 1$$

$$\text{Percentage of degree of saturation } (S_r) = \frac{0.004}{0.009} \times 100 = 44.4\%$$

$$\text{Percentage of water content } (w) = \frac{4}{25} \times 100 = 16\%$$

Example C2

The sample used for example C2 had the same weight of dry soil and void ratio as C1 but a different weight (32 kg). The water content and the degree of saturation are calculated as in example A1:

Weight of water (W_w) = 32 - 25 = 7 kg = 7 L water = 0.007 m^3 water = volume of water (V_w),

because the specific gravity of water is taken to be 1.0.

$$\text{Percentage of degree of saturation } (S_r) = \frac{0.007}{0.009} \times 100 = 77.8\%$$

$$\text{Percentage of water content } (w) = \frac{7}{25} \times 100 = 28\%$$

Example C3

The sample used for example C3 had the same weight of dry soil and void ratio as C1 and C2 but a different weight (34 kg). The water content and the degree of saturation are calculated as in example A1:

Weight of water (W_w) = 34 - 25 = 9 kg = 9 L water = 0.009 m^3 water = volume of water (V_w),

because the specific gravity of water is taken to be 1.0.

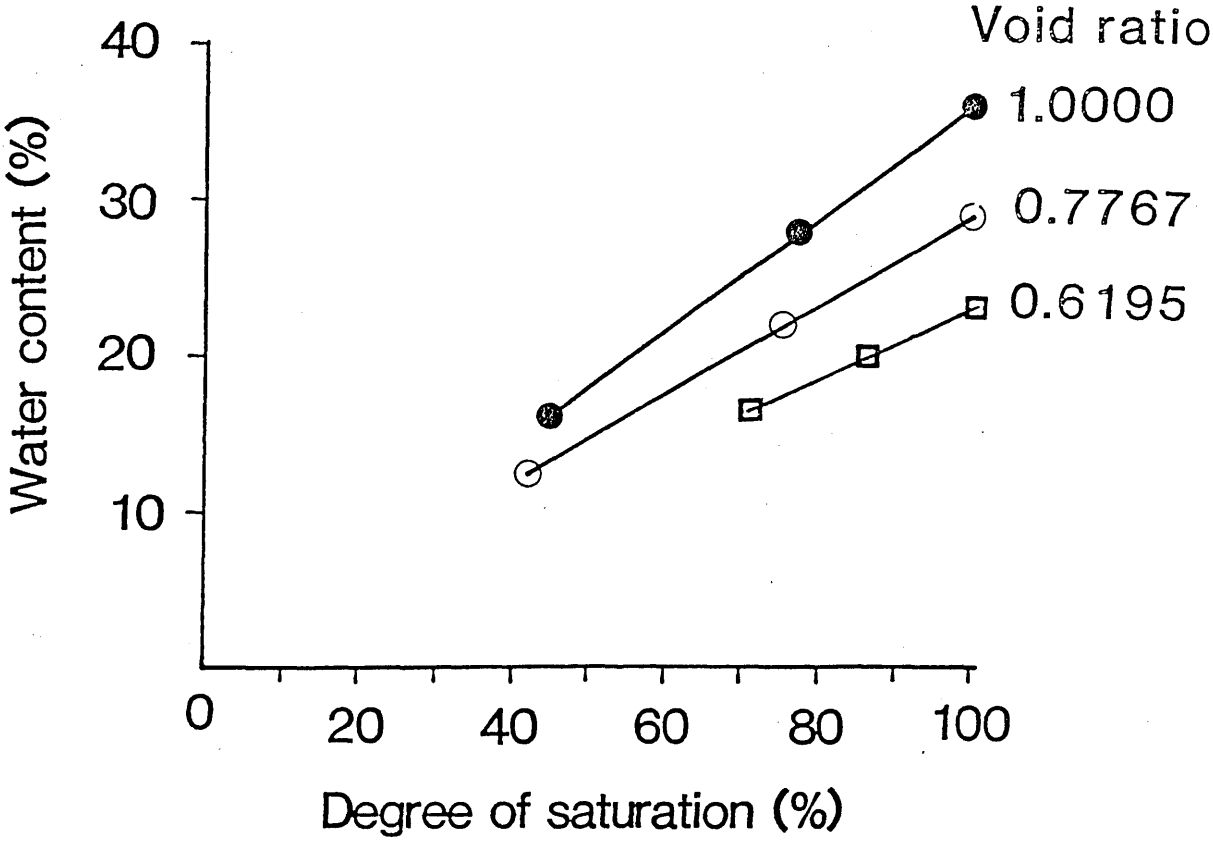
$$\text{Percentage of degree of saturation} = \frac{0.009}{0.009} \times 100 = 100\%$$

$$\text{Percentage of water content} = \frac{9}{25} \times 100 = 36\%$$

The results of the above calculations summarised in table 7, show that water content, degree of saturation and void ratio are different but related properties. For a given void ratio, as the water content increases the degree of saturation increases. This relationship is linear as can be seen in figure 18 where the data from table 7 are plotted. This figure also shows that an increase in void ratio produces an increase in water content and degree of saturation, in other words the slopes of the lines increase with increasing void ratio.

The graph (figure 18) suggests that there is an algebraic relationship between water content, degree of saturation and void ratio. I have been able to demonstrate that this is so in the following way.

Figure 18. The relationship between water content, degree of saturation and void ratio.


Void ratio
1.0000
0.7767
0.6195
Water content (%)
40
30
20
10
0
20
40
60
80
100
Degree of saturation (%)

From figure 18,

$$w = C\ S_r \qquad (16)$$

where,

w = water content,

S_r = degree of saturation,

C = a constant (the slope of the straight line).

This is a special example of the equation of a straight line $y = mx + c$, where $w = y$, $S_r = x$, and $C = m$. The intercept is at the origin, so $c = 0$.

From (16)

$$C = \frac{w}{S_r} \qquad (17)$$

Since

$$w = \frac{\text{weight of water } (W_w)}{\text{weight of dry solids } (w_s)}$$

and

$$S_r = \frac{\text{Volume of water } (V_w)}{\text{Volume of voids } (V_v)}$$

we can write C as

$$C = \frac{W_w}{W_s} \times \frac{V_v}{V_w} \qquad (18)$$

The volume of water in the sediment, V_w, can be written as (Smith 1981 p. 7):

$$V_w = \frac{W_w}{\gamma_w}$$

or

$$W_w = V_w\ \gamma_w \qquad (19)$$

The relationship between water content, degree of saturation and void ratio also can be obtained from equations 7, 9 and 10 as follows:

From equation 7,

$$W = \frac{Ww}{Ws} \qquad (7)$$

The volume of water in the sediment, Vw, can be written as:

$$Vw = \frac{Ww}{\gamma_w}$$

where γ_w = density of water

$$Ww = Vw\,\gamma_w \qquad (11)$$

and Vs, the volume of solids in a sediment can be written as:

$$Vs = \frac{Ws}{Gs\,\gamma_w}$$

where Gs = specific gravity

$$Ws = Vs\,Gs\,\gamma_w \qquad (12)$$

Substituting (11) and (12) in to (7) gives:

$$W = \frac{Ww}{Ws} = \frac{Vw\,\gamma_w}{Vs\,Gs\,\gamma_w} = \frac{Vw}{Vs\,Gs} \qquad (13)$$

Multipling both sides of equation (13) by $\frac{Vv}{Vw}$ gives

$$W \times \frac{Vv}{Vw} = \frac{Vw}{Vs\,Gs} \times \frac{Vv}{Vw}$$

Using equation (9) gives:

$$W \times \frac{1}{Sr} = \frac{Vv}{Vs} \times \frac{1}{Gs}$$

and equation (10):

$$\frac{W}{Sr} = \frac{1}{Gs} \times e$$

$$W\,Gs = Sr\,e$$

which is equation (22).

But $\frac{V_v}{V_s}$ = e the void ratio, hence

$$\frac{w}{S_r} = \frac{1}{G_s} \times e$$

Therefore,

$$w\, G_s = S_r\, e \qquad (22)$$

Equation 22 shows that there is an algebraic relationship between water content, degree of saturation and void ratio which can be written in full as:

water content (w) x Specific gravity (G_s) =

degree of saturation (S_r) x void ratio (e).

Note:

Equation 22 can also be obtained from equation 7, 9 and 10 as shown on the opposite page.

INTRODUCTION

Soils have many properties such as particle size, water content, permeability, and shear strength, which vary according to the type of soil. Water content is considered to be one of the most important. One of the effects of change in water content is a change in the permeability of the soil, which in turn is affected by the water potential. Water is held in the soil against gravitational forces draining water out and evaporation of water from the surface of the soil. The energy with which water is held in the soil at any water content can be specified as the soil-water potential or soil-water energy potential (Yong and Warkentin, 1975). Differences in potential energy of water between one point and another in a soil give rise to the tendency of water to flow within the soil. Soil water moves from where the potential energy is higher to where it is lower. It moves in the direction of decreasing potential energy. The rate of decrease of potential energy with horizontal distance ($-d\Psi/dD$) is the moving force causing flow (Hillel, 1971).

This section is divided into two parts. In the first, the definition of water potential and its component is discussed. The second part is concerned with the equations of a steady state water flow in a horizontal and vertical direction.

Water potential, ψ_w, (psi) is formally defined as the amount of work that a unit quantity of water in an equilibrium soil-water system is capable of doing when it moves to a pool of water in the reference state at the same temperature. The reference state commonly chosen is pure free water. The water movement to the reference state would have to occur through a semipermeable membrane, otherwise the solutes move with the water and it does not perform work (Baver, 1956; Harr, 1962; Hillel, 1971; Taylor and Ashcroft, 1972; Hanks and Ashcroft, 1980). A semipermeable membrane is a material that allows water but not salts to pass through it. In soil, air/water interfaces are almost perfect semipermeable membranes (Hanks and Ashcroft, 1980, pp. 50-51).

Water potential, ψ_w, is defined as

$$\psi_w = \psi_p + \psi_s + \psi_m \qquad (1)$$

in which

ψ_p = pressure potential

ψ_s = solute potential

ψ_m = matric potential.

These are defined later.

All of the potentials are defined with regard to a unit quantity of water, the units of the potential will depend on the way we specify the unit quantity of water. The units of potential corresponding to three methods for specifying a unit quantity of water are given below in the SI system.

1) If the quantity of water is expressed as a mass, the units of potential are ergs/g.

2) If the quantity of water is expressed as a volume, the units of potential are dynes/cm^3 (the same units as pressure).

3) If the quantity is expressed as a weight, the units of potential are cm of water.

Total Potential

Total potential, Ψ_t, is the amount of work that a unit quantity of water in an equilibrium soil water system is capable of doing when it moves to a pool of pure free water at the same temperature, located at a different reference level and subjected to atmospheric pressure. It is the sum of all potentials acting on water in an equilibrium system (Taylor and Ashcroft, 1972).

$$\Psi_t = \Psi_z + \Psi_w \qquad (2)$$

therefore

$$\Psi_t = \Psi_z + \Psi_m + \Psi_p + \Psi_s \qquad (3)$$

Because it is the sum of several terms, some of which may be positive or negative, the total potential may have either a positive or a negative value.

The components Ψ_z, Ψ_m, Ψ_p and Ψ_s are discussed in the following paragraphs. Total potential and hydraulic potential will also be defined.

Gravitational Potential

Gravitational potential, Ψ_z, is the energy associated with vertical position. It is the amount of work that a unit quantity of water in an equilibrium soil water system at an arbitrary level is capable of doing when it moves to another equilibrium system. This system will be identical in all respects except that it is at a different reference level (Taylor and Ashcroft, 1972). When the water

is above the reference level, it is capable of doing work under the acceleration of gravity and the gravitational potential is positive. When water is below the reference level, work must be performed in order to move it to the reference level and the gravitational potential is negative. The strength of gravitational potential depends on the force of gravity, the density of the material (in this case, water), and the elevation or vertical location of the system.

The gravitational potential, ψ_z, (Yong and Warkentin, 1975 p.109) is therefore

$$\psi_z = \rho_w g z \qquad (4)$$

in which

ρ_w = the density of water

g = the acceleration of gravity

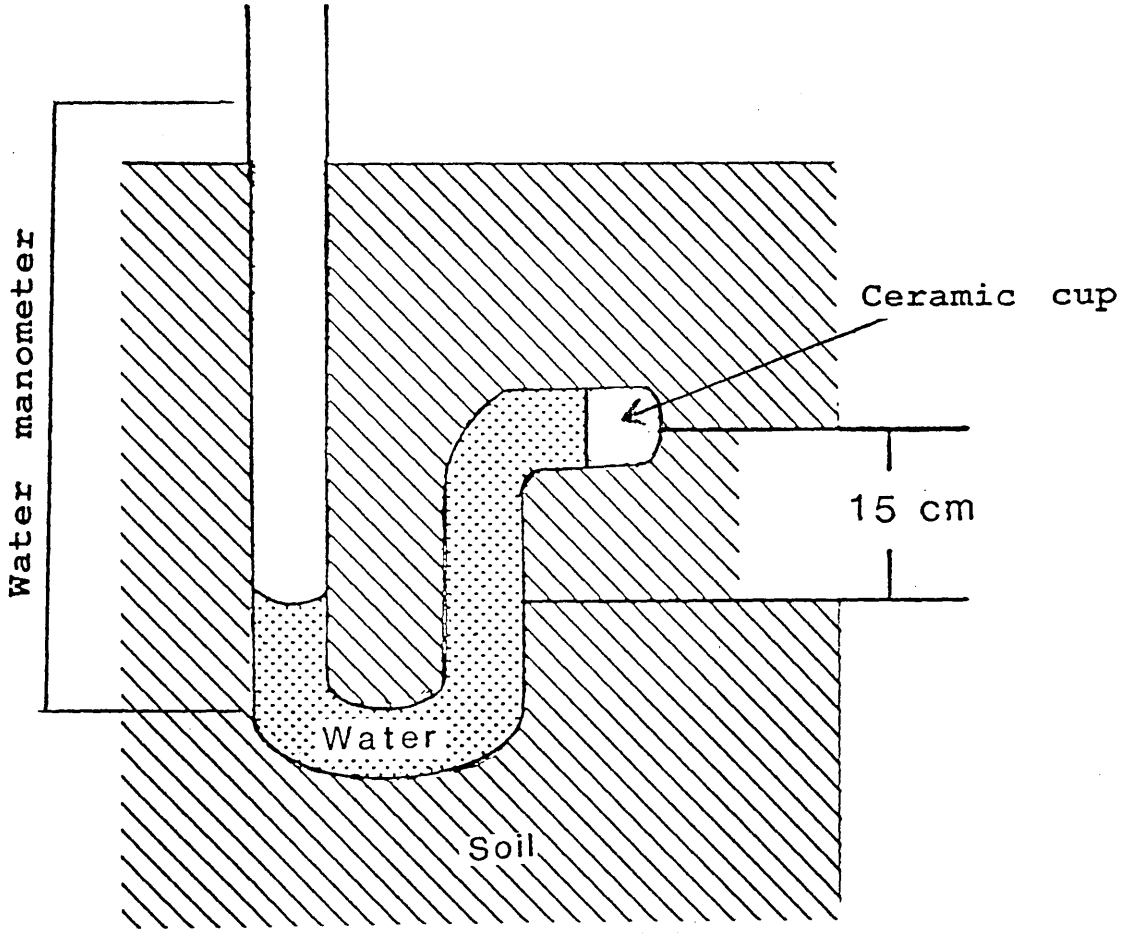
z = the distance measured in the vertical direction between the water mass and the reference level.

Matric Potential

The matric potential, ψ_m, is related to the adsorptive forces of the soil matrix (note: matric is the adjective derived from the noun matrix). If the unit quantity of water is expressed as a weight, then ψ_m at any given point is the vertical distance between that point in the soil and the water surface in a manometer filled with water and connected to the point in the soil in question via a ceramic cup (fig.1).

The matric potential is a dynamic property of soil. In saturated soil, ψ_m is zero. In theory, matric potential can be measured with the device (tensiometer) illustrated in figure 1 (Hanks and Ashcroft, 1980). In practice, one cannot get into the soil to install and take readings from this type of tensiometer. The commercially available instruments are, therefore, modified so that the water manometer is

Figure 1. An unglazed ceramic cup embedded in soil is connected to a water manometer to form a tensiometer. The weight matric potential of the soil water at the cup is the vertical distance from the centre of the cup to the water level in the manometer. For the situation illustrated, ψ_m = -15 cm (Hanks and Ashcroft, 1980).


Water manometer
Ceramic cup
15 cm
Water
Soil

replaced with a mercury manometer or with a vacuum gauge.

The following paragraphs describe the mercury manometer tensiometer and the vacuum gauge (Hanks and Ashcroft 1980). Concerning the mercury manometer tensiometer, a distance, z, is defined as the distance from the top of the mercury column to the centre of the ceramic cup (fig.2). A second distance, z_{Hg}, is defined as the distance from the top of the mercury column to the surface of the mercury in the reservoir.

For this situation, the weight matric potential, ψ_m, is defined by Taylor and Ashcroft (1972, p.286), as follows:

$$\psi_m = - z_{Hg} \frac{\rho_{Hg}}{\rho_w} + z \qquad (5)$$

in which

ρ_{Hg} = the density of mercury (13.6 g/cm^3)

ρ_w = the density of water (1.0 g/cm^3).

Substituting in equation 5 for the densities gives

$$\psi_m = - z_{Hg} \frac{13.6}{1.0} + z$$

Therefore

$$\psi_m = - 13.6\ z_{Hg} + z. \qquad (6)$$

The distance, z, varies as the height of the mercury column, z_{Hg}, changes. If, however, we consider the distance from the surface of the mercury reservoir to the centre of the cup, z_0, we have a constant for any given tensiometer. Substituting $z = z_0 + z_{Hg}$ into equation 5 gives

$$\psi_m = - z_{Hg} \frac{\rho_{Hg}}{\rho_w} + z_{Hg} + z_0 \qquad (7)$$

which can be written as

$$\psi_m = - z_{Hg} \left[\frac{\rho_{Hg}}{\rho_w} - 1\right] + z_0 \qquad (8)$$

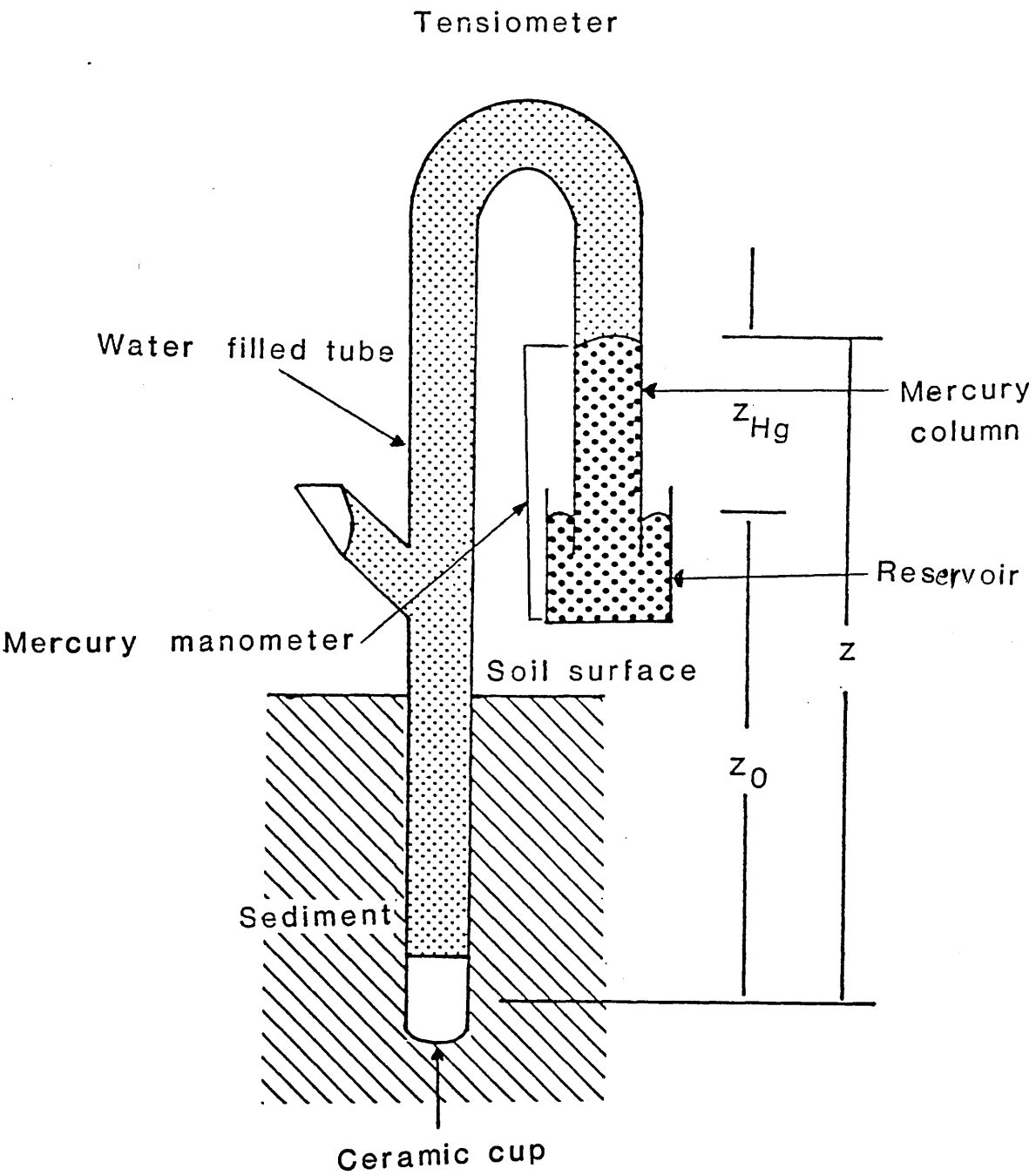
Figure 2. A tensiometer constructed by connecting a ceramic cup to a mercury manometer via a water-filled tube. The illustrated symbols refer to equation 6 and 9 which can be used to compute matric potential, ψ_m. The definitions of the symbols are as follows:

z = the distance from the top of the mercury column to the centre of the ceramic cup

z_{Hg} = the distance from the top of the mercury column to the surface of the mercury in the reservoir

z_0 = the distance from the surface of the mercury reservoir to the centre of the cup.

(Hanks and Ashcroft, 1980).


Tensiometer
Water filled tube
Mercury manometer
Soil surface
Sediment
Ceramic cup
z_{Hg}
Mercury column
Reservoir
z
z_0

Substituting for the densities gives

$$\psi_m = - z_{Hg} \left[\frac{13.6}{1.0} - 1 \right] + z_0$$

Therefore

$$\psi_m = - 12.6 \, z_{Hg} + z_0 \qquad (9)$$

Equation 5 has a positive value of z when the mercury level in the manometer is above the tensiometer cup. On the other hand when the mercury level in the manometer is below the tensiometer cup, z, has a negative value and the equation will be as follows

$$\psi_m = - z_{Hg} \frac{\rho_{Hg}}{\rho_w} - z_0 \qquad (10)$$

In the vacuum gauge tensiometer, the mercury is replaced by a vacuum gauge. The reading on the dial can be converted to ψ_m (weight quantity) in a way similar to that used with the mercury manometer. One should be very careful to determine the units in which the gauge is calibrated.

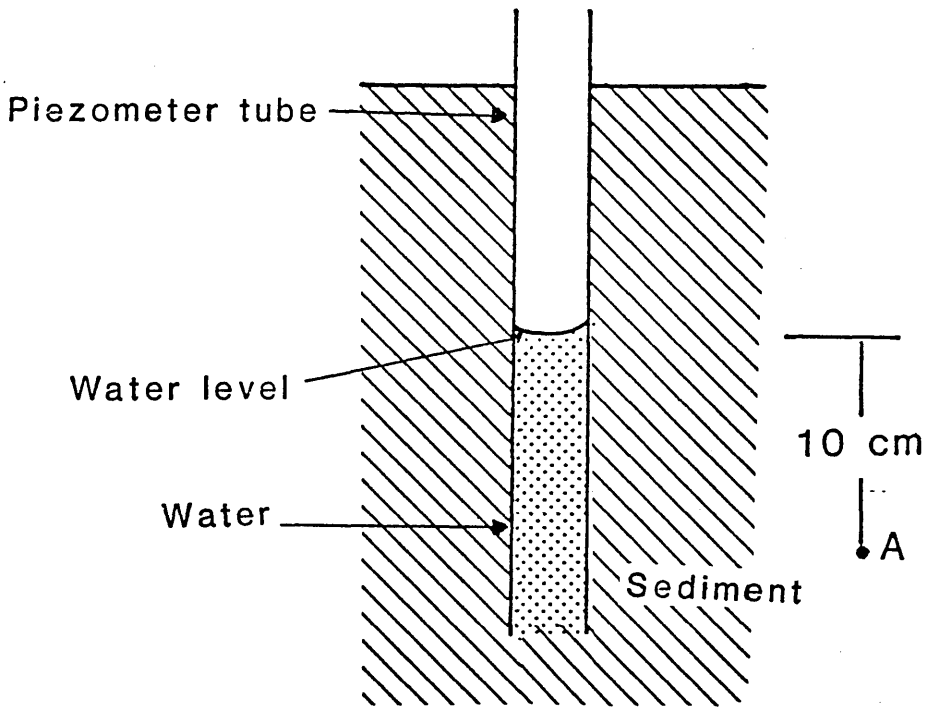
Most commercially available gauge tensiometers have a dial calibrated from 0 to 100 which in weight matric potential units corresponds to a range of 0 to -1000 cm. Some tensiometers may be calibrated from 0 to 1000 to cover the 0 to -1000 cm range (Taylor and Ashcroft, 1972, p.160 ; Hanks and Ashcroft, 1980, pp. 22-25).

Pressure Potential

The pressure potential, ψ_p, under field conditions applies mostly to saturated soil. If the quantity of water is expressed as a weight, then ψ_p is the vertical distance from the point in question in the soil to the water surface of a piezometer connected to the point in question, in other words to the water table in the soil (see fig.3).

Figure 3. A piezometer tube which is used to determine the level of the water table in the soil and can also be used to determine the pressure potential of soil water. The pressure potential of any point in the soil is the distance between the point and the water level in the piezometer tube. Thus, the pressure potential at point A is

Ψ_p = 10 cm (Hanks and Ashcroft, 1980).


Piezometer tube
Water level
Water
Sediment
10 cm
A

In the field, ψ_p is zero above and at level of the water in the piezometer. Below this level, ψ_p becomes positive. It increases with depth below this water level though the water content of the soil does not change. This component of the pressure potential, which is the only component normally existing under field conditions, is called the submergence potential, ψ_u. Thus in the field, ψ_p, is measured with a piezometer which measures submergence potential.

In the laboratory, pressure potential is measured using a piezometer (fig. 3), which is a tube, open at both ends, that is placed in the soil. Water flows into the bottom of the tube to a level which defines the water table. The water level in the tube is usually determined with an electrical device. At the water surface and all points above, ψ_p is zero. Below the water surface, ψ_p at any point is equal to the depth below the water surface of the point in question (the depth between the water surface and the point in question) (Taylor and Ashcroft, 1972; Hanks and Ashcroft, 1980, p.26).

Solute Potential

Solute potential, ψ_s, is a result of soluble materials, such as salts, in the soil solution and arises because of these and the presence of a semipermeable membrane in the system. In the soil water system there are two important semipermeable membranes:

1) Cell Walls in roots - these are not perfect semipermeable membranes as some salts pass into the roots.

2) Air water interfaces - near perfect semipermeable membranes.

The solute potential in soil is relatively unimportant in liquid water flow because there are no semipermeable membranes to restrict the passage of solutes. However it is of great importance to plants

as plants have semipermeable membranes.

For example, if a plant is growing in pure fresh water (no salt molecules), solute potential has a large effect because water flow into the roots is not restricted by the semipermeable root membrane. If plants are growing in a salty moist soil, solute potential has less effect because water flow into the roots is more restricted through semipermeable membranes.

We are often interested in the salt concentration that produces a given solute potential. An approximation is given by

$$\Psi_S = - R T C_s \qquad (11)$$

where

Ψ_S = solute potential

R = the universal gas constant (82 bars cm^3/mol K)

T = absolute temperature (K)

C_s = solute concentration (must be in mol/cm^3 with the above units for the gas constant).

An exact value of C_s is often difficult to determine because it is the summation of all species including the dissociation into ionic species (Hanks and Ashcroft, 1980, pp. 50-51).

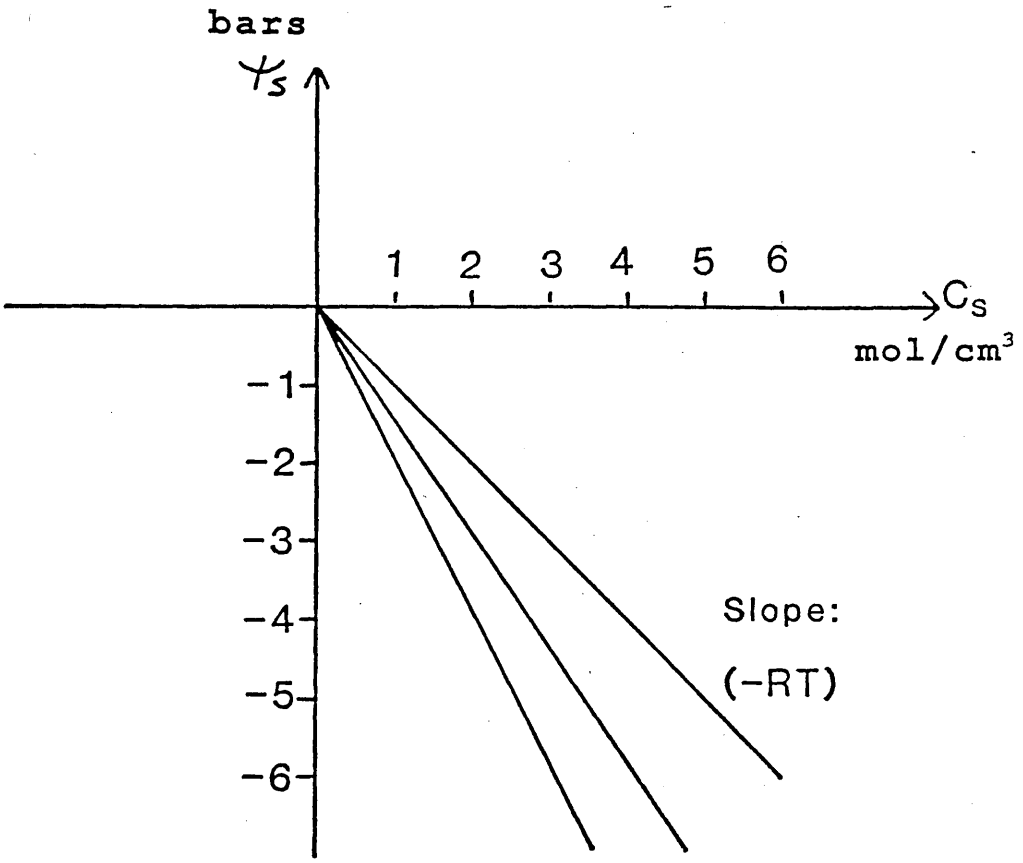
Looking at equation 11 helps us to understand that solute potential is always negative or 0, and an increase in C_s gives a decrease in solute potential (fig.4).

The change in solute potential is proportional to the change in C_s and the multiple (-RT) is the proportionality factor (the slope of the straight line which is negative slope).

Hydraulic potential

To determine in which direction water will flow, we have to know the total water potential at different points in the system. Under

figure 4. The x axis represents the solute concentration (Cs), the y axis represents the solute potential (ψ_s). Solute potential is always negative or 0, and an increase in solute concentration ($C_{s)}$ gives a decrease in solute potential. The multiple (-RT) is the slope of the straight line which is the negative slope where R is the universal gas constant and T is the absolute temperature (See Hanks and Ashcroft, 1980 p.51).


bars
ψ_s
1
2
3
4
5
6
C_s
mol/cm^3
-1
-2
-3
-4
-5
-6
Slope:
(-RT)

isothermal conditions, water flows from locations where the total water potential is high to locations where the total water potential is low.

If we are concerned with liquid water flow in the soil, where there is no semipermeable membrane, the solute potential is zero (solute potential does not act as a driving force in water flow). Thus, for liquid flow

$$\Psi_t = \Psi_z + \Psi_m + \Psi_p.$$

This combination of potentials is called the hydraulic potential, (Harr, 1962; Luthin, 1966; Cedergren, 1977; Hillel, 1971; Taylor and Ashcroft, 1972; Dunn, et al. 1980; Hanks and Ashcroft, 1980).

Consequently, the hydraulic potential, Ψ_h, is

$$\Psi_h = \Psi_z + \Psi_m + \Psi_p.$$

For equilibrium conditions, the hydraulic potential is everywhere constant and there is no water flow (Taylor and Ashcroft, 1972; Hanks and Ashcroft, 1980).

Liquid water flows as a result of a hydraulic potential gradient. Thus Ψ_h will vary throughout the parts of the soil in which flow is occurs (Hanks and Ashcroft, 1980).

INTRODUCTION

Horizontal and vertical flow in relation to the concepts outlined above will be discussed in this section. This problem has been dealt with, amongst many others, by Taylor and Ashcroft (1972), Yong and Warkentin (1975), Rushton and Redshaw (1979), and Hanks and Ashcroft (1980). My account is largely based on Hanks and Ashcroft (1980), Taylor and Ashcroft (1972), and Hillel (1971).

In steady state flow, flow characteristics do not change with time, although they may change with location. Some examples of flow characteristics that do not change with time (Hanks and Ashcroft, 1980) are:

1) Water content at each point in the flow system.

2) Hydraulic potential (and all of its component potentials).

3) Water flux through any area.

Horizontal Flow

The rate of flow will depend on the rate of decrease of potential energy in a horizontal direction. This rate is also called the hydraulic potential gradient (Taylor and Ashcroft 1972, p.187). The general equation for horizontal liquid flow is:

$$J_w = - k \frac{\Delta \Psi_h}{\Delta s} \qquad (12)$$

in which

J_w = water flux density (cm/s)

k = permeability cofficient (cm/s)

$\Delta \Psi_h$ = the difference in hydraulic potential (cm) between two points separated by a distance Δs (cm) where s is a

horizontal distance measured along the direction of flow.

Soil water moves constantly in the direction of decreasing hydraulic potential. The rate of decrease of hydraulic potential with horizontal distance is the moving force causing flow ($-\Delta\Psi_h / \Delta s$) (Hillel 1971, and Cedergren, 1977). The negative sign indicates that the force acts in the direction of decreasing potential.

The proportionality constant k, in equation 12, has been given various names: hydraulic conductivity, permeability coefficient, conductivity, transmission coefficient, transmissivity, capillary conductivity and permeation coefficient (Taylor and Ashcroft, 1972, p.187). Permeability and hydraulic conductivity are two of the most commonly used terms in the literature and the ones which are used in other sections of this thesis.

Vertical Flow

For vertical flow the equation is:

$$J_w = - k \frac{\Delta\Psi_h}{\Delta z} \qquad (13)$$

where

z = the distance (cm) measured in the vertical direction.

The quantity per unit area per unit time, Q/At, is called the flux density, Jw (Hanks and Ashcroft, 1980 p63 and Taylor and Ashcroft, 1972, p.187). Thus equation 13 can be written as

$$J_w = \frac{Q_w}{At} = - k \frac{\Delta\Psi_h}{\Delta z} \qquad (14)$$

in which

Q_w = the quantity of flowing water

A = area which the water flows through

t = time that quantity Qw takes to flow.

* "At a free surface, both the matric potential, ψ_m, and the pressure potential, ψ_p, are zero. We should also note that ψ_m and ψ_p must be zero at the very bottom of the column before water will drip from the lower end (actually ψ_p must be just slightly greater than zero before water will drip) (Hanks and Ashcroft, 1980, p.37,40,64).

Upward flow is taken as positive and downword flow as negative.

Figure 5 shows Darcy's original experiment for measuring permeability in a saturated soil. The cross sectional area is 100 cm^2 and there were 500 cm^3 of water collected in the container in 10 hours. To find the saturated hydraulic conductivity we proceed as follows.

Flow is downward; therefore Q is negative ($Q = -500\ cm^3$). The hydraulic potential at point A is the sum of pressure potential and gravitational potential at A, because the matric potential is zero (the material is saturated).

Therefore, the hydraulic potential at point A is

$$\Psi_{hA} = \Psi_{pA} + \Psi_{zA} = 12\ cm + 15\ cm = 27\ cm,$$

and at point B is

$$\Psi_{hB} = \Psi_{pB} + \Psi_{zB} = 0\ cm + 0\ cm = 0\ cm, \quad *$$

$$\frac{\Delta\Psi_h}{z} = \frac{\Psi_{hA} - \Psi_{hB}}{z_A - z_B} = \frac{27\ cm - 0\ cm}{0\ cm - (-15\ cm)} = \frac{27\ cm}{15\ cm} = 1.8$$

Substituting in equation 14 we obtain

$$k = -\frac{Q_w}{At}\frac{\Delta z}{\Delta\Psi_h} = -\frac{-500\ cm^3}{(100\ cm^2)\ (10\ h)\ (3.6 \times 10^3 s/h)} \times \frac{1}{1.8} =$$

$$k = 7.7 \times 10^{-5}\ cm/s.$$

When soil is not saturated, hydraulic conductivity (permeability), k, is highly dependent on the soil water content. There is a very large increase in hydraulic conductivity with the increase in soil water content. Table 1 (Hanks 1965; Hanks and Ashcroft 1980) shows these increases, and the increase from highly negative values to zero in the matric potential.

Figure 5. Darcy's original experiment for measuring permeability in saturated soil. The cross sectional area is 100 cm^2 and 500 cm^3 of water collected in the container in 10 hours. Flow of water is downward, the pressure potential at point A equals 12 cm and the gravitational potential equals 15 cm. Thus the hydraulic potential at point A equals 27 cm. The pressure potential at point B equals 0 cm, the gravitational potential also equals 0 cm, thus the hydraulic potential equals 0 cm.

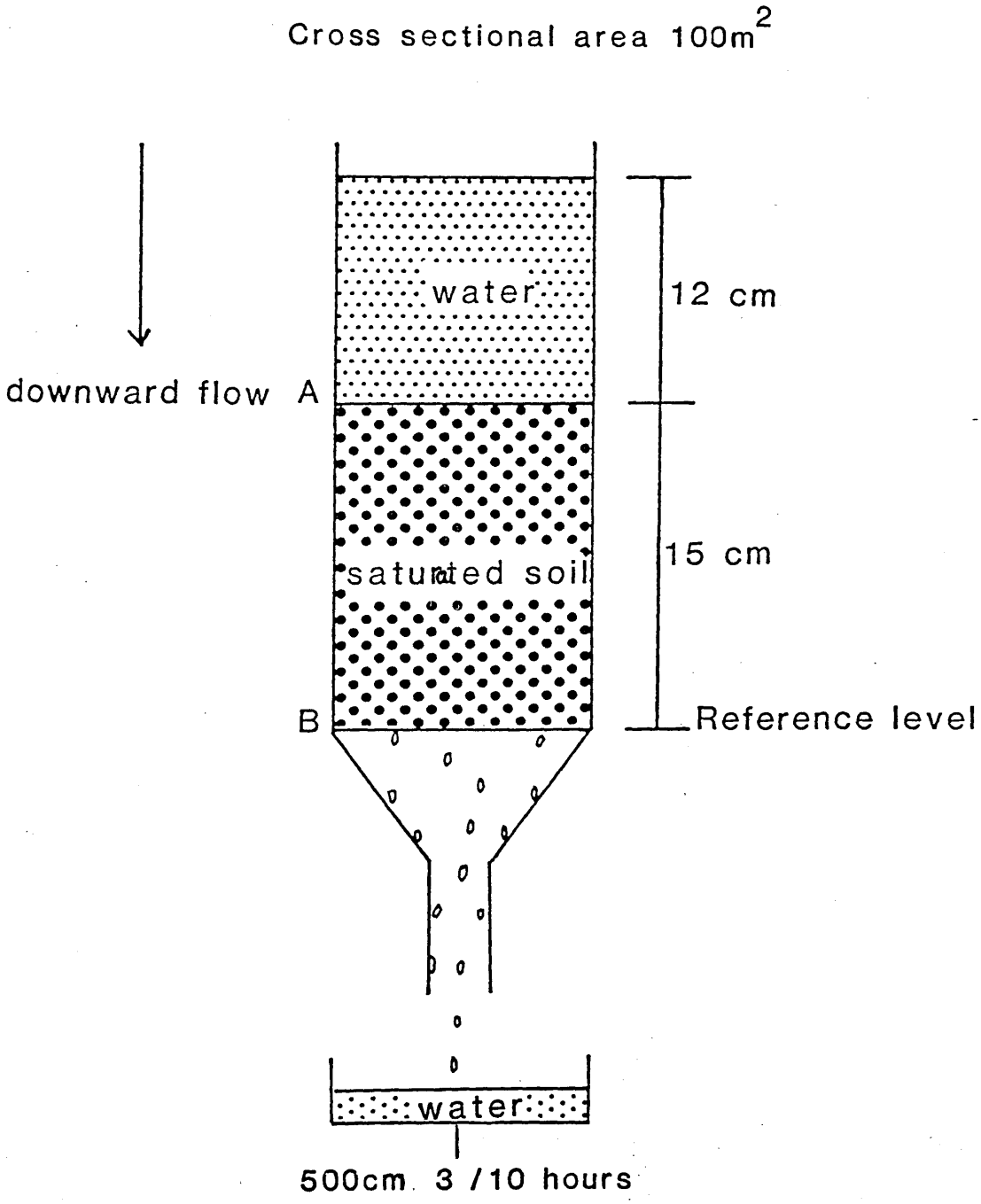

Cross sectional area 100m2
downward flow
A
water
12 cm
saturated soil
15 cm
B
Reference level
water
500cm 3 /10 hours

Table 1. The hydraulic conductivity and matric potential of two soils at several water contents. There is a very large decrease in hydraulic conductivity with the decrease in soil water content and also a decrease in matric potential in the same way with water content (Hanks, 1965; Hanks & Ashcroft, 1980).

Volume water content	Sarpy loam		Geary silt loam	
	hydraulic conductivity K_w (cm/day)	matric potential (cm)	hydraulic conductivity K_w (cm/day)	matric potential (cm)
0.05	4.5×10^{-5}	-6.975	_	_
0.06	6.7×10^{-5}	-3.365	_	_
0.08	4.1×10^{-4}	-1.255	-	-
0.10	4.8×10^{-3}	-447	_	_
0.12	2.6×10^{-2}	-330	_	_
0.14	5.2×10^{-2}	-259	_	_
0.16	7.8×10^{-2}	-209	-	-
0.18	1.1×10^{-1}	-168	6.4×10^{-5}	-7.685
0.20	2.7×10^{-1}	-134	4.1×10^{-4}	-4.025
0.22	7.4×10^{-1}	-106	2.0×10^{-3}	-2.675
0.24	1.6	-78	3.6×10^{-3}	-1.675
0.26	3.6	-64	1.6×10^{-2}	-815
0.28	4.7	-53	4.5×10^{-2}	-525
0.30	7.4	-43	1.1×10^{-1}	-331
0.32	1.1×10	-34	2.8×10^{-1}	-212
0.34	1.9×10	-26	5.4×10^{-1}	-143
0.36	3.4×10	-18	1.0	-94
0.38	6.9×10	-10	2.1	-59
0.40	1.1×10^2	-3	4.1	-36
0.41	1.2×10^2	0	5.1	-28
0.42	_	_	6.2	-21
0.44	_	_	7.3	-10
0.46	_	_	9.5	0

One method of evaluating hydraulic conductivity of a soil as a function of water content is to establish steady state flow with a water table (Hanks and Ashcroft, 1980). Steady state flow is established by sprinkling the soil with water at an application rate that is insufficient to cause water ponding on the soil surface. When the steady state is achieved, both the water content and the matric potential are uniform in the upper portion of the soil profile. As to be expected, at depths approaching the water table they both approach their saturation values. The matric potential and water content are then measured in the upper portion of the soil where they are uniform (i.e. where they do not change with depth).

In the zone of constant matric potential, $\Delta\psi_m$ is zero and consequently $\Delta\psi_h = \Delta\psi_z$. That is, the flow is entirely a result of the gravitational potential gradient ($\frac{\Delta\psi_z}{\Delta z}$). The gravitational potential difference ($\Delta\psi_z$) for vertical flow is always equal to the depth difference (Δz).

Hence

$$\Delta\psi_z = \Delta z$$

and

$$\Delta\psi_h = \Delta\psi_z$$

therefore

$$\Delta\psi_h = \Delta\psi_z = \Delta z$$

and hence the gravitational gradient for vertical flow ($\frac{\Delta\psi_z}{\Delta z}$) equals 1. Therefore

$$\frac{\Delta\psi_z}{\Delta z} = \frac{\Delta\psi_h}{\Delta z} = 1. \qquad (15)$$

From equation 13 and 15, the value of the permeability (in the zone of constant matric potential and also constant water content) is numerically equal to the flux density of water application ($k = -J_w$) (Hanks and Ashcroft, 1980).

In the zone of uniform matric potential, the steady state matric potential depends on the application rate. Thus, by using different application rates, hydraulic conductivities can be established for various matric potentials (or water contents) (Hanks and Ashcroft, 1980). Typical results are shown in table 1 (Hanks 1965).

Hanks and Ashcroft (1980 p.66) give a worked example of calculating hydraulic conductivity which I have paraphrased below as follows. Initially dry loam soil was used in the experiment. Loam is a rich soil containing clay, sand and organic matter. Water was allowed to flow through the soil to a water table at - 100 cm. Water flux density (11 cm/day), the matric potential and the water content remain constant in the flow system (steady state flow). To find the hydraulic conductivity of the soil for the zone between -10 cm and -40 cm, where the matric potential is constant, we proceed as follows.

$$J_w = -11 \text{ cm/day}$$

where J_w is the flux density. Since the flow is downward, the flux density is negative. J_w is constant throughout the profile because steady state flow.

Since

$$k = -J_w,$$

$$k = - (-11 \text{ cm/day}) = 11 \text{ cm/day}.$$

where k is the hydraulic conductivity, as required.

APPENDIX 1.1

ELECTRICAL ANALOGUE OF WATER FLOW THROUGH SEDIMENTS OHM'S LAW AND DARCY'S LAW

Ohm's law, gives the fundamental relation for flow of an electric current, and it is given by the following equation:

$$I = \frac{E}{R} \qquad (1)$$

where,

I = current in amperes. One ampere is the current required to move one coulomb of charge per time past a fixed part in a conductor and it is therefore a unit of flow (Pitt 1977).

E = pressure in volts, a potential function.

R = resistance in Ohms.

Ohm's law has been used in soil physics to solve problems of water movement through soil by electric analogues (Kirkham and Powers, 1972, p.75).

Ohm's law and Darcy's law have similarities and form the basis for the electrical analogue for groundwater flow problems (analogue means similarity of properties or relations, without identity).

Ohm's law is expressed in terms of resistance, whereas Darcy's law is in terms of conductivity. Since in Ohm's law, the conductivity K' is the reciprocal of resistance,

$$K' = \frac{1}{R},$$

and equation 1 can be written as;

$$I = K' E \qquad (2)$$

The conductance K' varies in direct relationship with the specific conductivity k', which is the conductance of the material between opposite sides of a cube, one centimetre in all dimensions. K' also varies in a directed relationship with the area, A, but only inversely with the length, L. The equation is as follows:

$$K' = k' \frac{A}{L}$$

Therefore, equation 2 can be written as:

$$I = k' \frac{E}{L} A \qquad (3)$$

Since I is the quantity of flow, k' the specific conductivity, $\frac{E}{L}$ the voltage gradient, and A the flow area, it may be seen that equation 3 is similar to Darcy's equation which is

$$Q = K \frac{H}{L} A$$

where,

Q = the quantity of water flowing per unit time

K = the hydraulic conductivity (permeability coefficient)

$\frac{H}{L}$ = hydraulic gradient

A = cross-sectional flow area.

Thus, the flow of electricity through the conducting medium may be used as a model for the flow of water through soil. Two types of conducting medium are commonly used: conducting paper and a resistance network (Herbert and Rushton, 1966).

Slichter (1897) was one of the first who recognised the analogy between electrical flow and groundwater flow (Luthin, 1966).

Childs (1943; 1945a; 1945b; 1946; 1947) and Childs and O'Donell (1951) have published a series of solutions to flow problems using electric analogues, having been stimulated to do so by Wyckof and Reed's (1935) early work. Childs (loc. cit.) has drawn flow nets for regions of different drain tile spacings for subsurface drainage. This includes flow nets for different heights of tiles above an impermeable layer or flow regions with a capillary fringe, and for the unsteady state flow associated with a rising and falling water table. Childs and Collis-George (1950) have made extensive use of the relation between Ohm's law and Darcy's law in their analysis of a wide variety of groundwater problems. They used an electrical analogue made by soaking sheets of filter paper in graphite. According to Luthin (1953, 1966) the conductive paper is available commercially under the name of Teledeltos. Kemper and Lutz (1956) used an electric analogue to evaluate the effect of cracks, old root holes, and worm holes on the measurement of hydraulic conductivity of soil. Luthin (1953) and Bouwer and Little (1959) used a network of electrical resistances to build an electrical analogue. By changing the resistance of certain resistors they were able to simulate soil layers with different hydraulic conductivities, and they obtained flow nets for stratified soils. Bouwer and Little (1959) used an electrical resistance network to account for the capillary fringe in drainage problems.

APPENDIX 1.2

KINEMATIC VISCOSITY, VISCOSITY AND DENSITY

This section describes kinematic viscosity. Kinematic viscosity is calculated from viscosity and density and so I shall also describe viscosity and density.

In the majority of problems involving viscosity, we are concerned with the magnitude of the viscosity compared with the magnitude of the inertia forces, that is, those forces causing acceleration of particles of the fluid. The ratio of viscosity (μ) to density (ρ) is known as the kinematic viscosity and denoted by the symbol (ν) so that

$$\nu = \frac{\mu}{\rho}$$

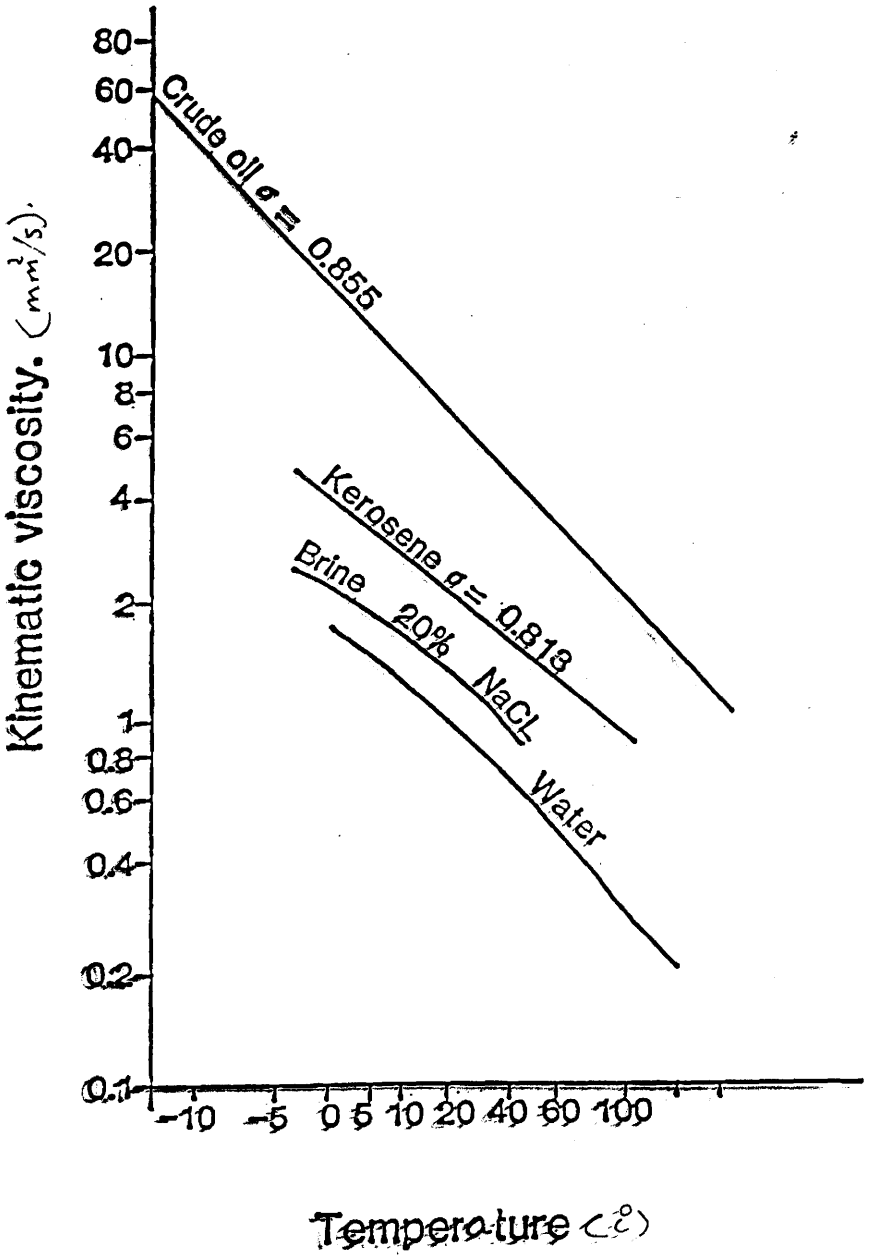
The dimensional formula of ν is given by

$$\frac{\dfrac{M}{LT}}{\dfrac{M}{L^3}} = \frac{L^2}{T}$$

Hence the magnitudes of length and time are involved.

The basic unit of kinematic viscosity m^2/s is too large for most purposes and so mm^2/s ($= 10^{-6}\ m^2/s$) is generally used. The CGS unit, cm^2/s, is termed the stokes, after the Cambridge physicist, Sir George Stokes (1819 - 1903), who contributed much to the theory of viscous fluids. This unit is rather large, and so the centistoke (cS) (i.e. 10^{-2} stokes = mm^2/s) is widely used. Figure 1 shows that the kinematic viscosity of water, brine (20% NaCl), kerosen ($\sigma = 0.813$) and crude oil ($\sigma = 0.853$) all decrease with increasing temperature (Massey, 1983).

Figure 1. The kinematic viscosity of water, brine 20% NaCL, Kerosen $\sigma = 0.813$ and Crude oil $\sigma = 0.853$ all decrease with increasing temperature (Massey, 1983 page 20).


Kinematic viscosity. (mm²/s).
80
60
40
20
10
8
6
4
2
1
0.8
0.6
0.4
0.2
0.1
Crude oil ρ = 0.855
Kerosene ρ = 0.818
Brine 20% NaCl
Water
-10 -5 0 5 10 20 40 60 100
Temperature (°)

Viscosity and its units

All real fluids resist any force which tends to cause one layer to move over another. This resistance occurs only while the movement is taking place. Thus when the external force is removed, the flow subsides because of the resisting forces. When the flow stops, the particles of fluid stay in the positions they have reached and have no tendency to revert to their original positions. This resistance to the movement of one layer of fluid over an adjacent one is caused by the viscosity of the fluid. Since relative motion between layers requires shearing forces, that is, forces parallel to the surfaces over which they act, the resisting forces must be in exactly opposite directions and so they too are parallel to the surfaces.

The magnitude of the resisting, that is viscous, forces depends not only on external conditions but on the nature of the fluid itself. In other words under particular conditions one fluid offers greater resistance to flow than another. Liquids such as glycerine and crude oil cannot be rapidly poured or easily stirred, and are often called "thick". Thin liquids such as water and petrol flow much more easily. Gases as well as liquids have a viscosity, although their viscosity is much lower. The relationship of permeability varies inversely as the viscosity, which increases with a decrease in temperature, has been found to be true (Wallace, 1948).

Quantitative Definition of Viscosity

If we consider the motion of fluid as all the particles moving in the same direction, but with different layers of the fluid moving at different velocities, then one layer moves faster or slower relative to another layer.

The rate at which the velocity changes with the distance across the flow is called the velocity gradient. Suppose that the velocity u varies with distance y (Fig 2). The velocity gradient is given by $\frac{\delta u}{\delta y}$ or, in the limit as $y \rightarrow 0$, by $\frac{\delta u}{\delta y}$. The partial derivative $\frac{\delta u}{\delta y}$ is used because in general the velocity also varies in other directions. Figure 3, shows two adjacent layers of the fluid slightly separated for clarity. If the upper layer is the faster of the two it will pull the lower one with it by means of a force F acting on the lower layer. At the same time, the lower layer tends to retard the faster upper one by an equal and opposite force (Newton's third law). If the force F acts over an area of contact A the stress τ is given by $\frac{F}{A}$. Newton (1642 - 1727) postulated that, for the straight and parallel motion of a given fluid, the tangential stress between two adjacent layers is proportional to the velocity gradient in a direction perpendicular to the layer. That is

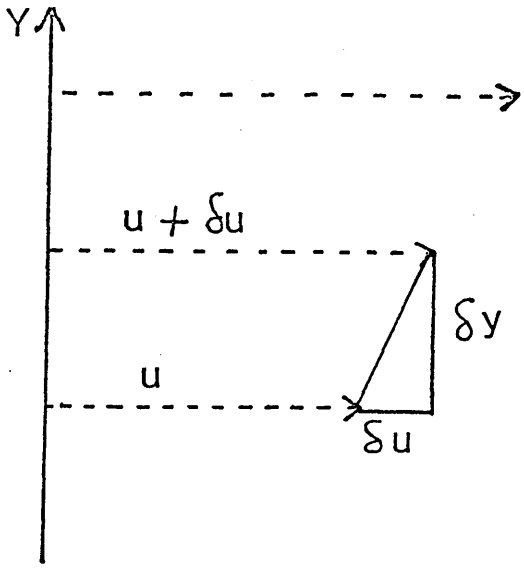
$$\tau = \frac{F}{A} \propto \frac{\delta u}{\delta y} \qquad \text{or} \qquad \tau = \mu \frac{\delta u}{\delta y} \qquad (1)$$

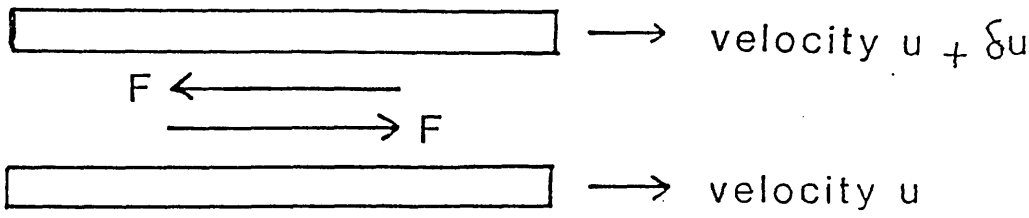
Where μ is a constant for a particular fluid at a particular temperature. This coefficient of proportionality (μ) is the coefficient of viscosity, that is, the viscosity of the fluid.

Viscosity μ is a property of the fluid and a scalar quantity. The other terms in equation 1, however, refer to vector quantities, and it is important to define their directions. For many fluids the magnitude of the viscosity is independent of the rate of shear, and although it may vary considerably with temperature it is constant for a particular

Figure 2. The motion of the fluid as all the particles moving in the same direction, but with different layers of the fluid moving at different velocities, then one layer moves faster or slower relative to another layer. The rate at which the velocity changes with the distance across the flow is called the velocity gradient $\left(\frac{\delta u}{\delta y}\right)$. u is velocity and y is distance (measured from fixed reference plane) (Massey, 1983).

Figure 3. Two adjacent layers of the fluid slightly separated for clarity. The upper layer is the faster of the two and it pulls the lower one with it by means of the force F acting on the lower layer. At the same time, the lower layer tends to retard the faster upper one by an equal and opposite force (Massey, 1983).


Y
u + δu
δy
u
δu


velocity u + δu
F
F
velocity u

fluid and temperature. Equation 1, shows that, irrespective of the magnitude of μ, the stress is zero when there is no relative motion between adjacent layers. Moreover it is clear from the equation that $\frac{\delta u}{\delta y}$ must not be infinite, since this would cause an infinite stress which is physically impossible. Hence if the velocity varies across the flow, it must do so continuously and not change abruptly between adjoining elements of the fluid. This condition of continuous variation must also be met at a solid boundary, the fluid immediately in contact with the boundary does not move relative to it because such motion would constitute an abrupt change.

Viscosity is defined as the ratio of a shear stress to a velocity gradient. Shear stress is defined as the ratio of a force to the area over which it acts (i.e. force per unit area). Its dimensional formula is F/L^2. A velocity gradient is defined as the ratio of increase of velocity to the distance across which the increase occurs, thus giving the dimensional formula

$$\frac{L/T}{L} = \frac{1}{T}$$

The dimensional formula of viscosity $\left(\frac{\text{shear stress}}{\text{velocity gradient}}\right)$ is therefore

$$\frac{F/L^2}{1/T} = \frac{FT}{L^2}$$

Since

$$F = \frac{ML}{T^2}$$

the expression is equivalent to $\frac{M}{LT}$. Therefore the unit of viscosity is the force unit X time divided by area unit, such as dyne X second/cm^2. Smaller units, the centipoise, cp,i.e.10^{-2} poise, the

millipoise, mp (10^{-3} poise) and the micropoise, μ (10^{-6} poise) are also used.

Density and its units

Density at a point (ρ), is the limit to which the mean density approaches as volume is indefinitely reduced, that is $\lim_{v \to 0}$ (m/V). As a mathematical definition this is satisfactory. However all matter consists of separate molecules, so we should think of the volume reduced not to absolute to zero, but to very small amount that is nevertheless large enough to contain many molecules. Its dimensional formula is

$$\frac{M}{L^3}$$

However the mean density ($\bar{\rho}$), is the ratio of the mass of a given amount of a substance to the volume which this amount occupies. If the mean density in all parts of a substance is the same, then the density is said to be uniform. Its dimensional formula is

$$\frac{M}{L^3}$$

Relative density (specific gravity, σ), is the ratio of the density of a substance to some standard density. For solids and liquids, the standard density chosen is the maximum density of water. This occurs at 4^{o} C. For gases, the standard density is that of air or hydrogen, although the term is little used for gases. As relative density is the ratio of two magnitudes of the same kind, it has no units.

A P P E N D I X T W O

Reading	Coefficient of permeability	degree of saturation
1	4.63×10^{-3}	86.0
2	4.69×10^{-3}	86.3
3	4.76×10^{-3}	87.1
4	5.22×10^{-3}	89.7
5	5.76×10^{-3}	92.0
6	6.78×10^{-3}	96.0
7	7.49×10^{-3}	98.4
8	7.76×10^{-3}	99.5
9	7.75×10^{-3}	100

Appendix 2; Table 1. Wallace's (1948) data of Union Falls A.

Reading	Coefficient of permeability	Degree of saturation
1	3.56×10^{-3}	83.6
2	3.61×10^{-3}	83.8
3	3.64×10^{-3}	84.4
4	3.79×10^{-3}	86.0
5	4.27×10^{-3}	88.8
6	4.74×10^{-3}	91.4
7	5.18×10^{-3}	93.3
8	6.14×10^{-3}	97.0
9	6.60×10^{-3}	98.8
10	6.73×10^{-3}	100
11	6.80×10^{-3}	100

Appendix 2; Table 2. Wallace's (1948) data of Union Falls B.

Reading	Coefficient of permeability	degree of saturation
1	2.23×10^{-3}	78.3
2	2.20×10^{-3}	78.4
3	2.22×10^{-3}	79.2
4	2.42×10^{-3}	81.9
5	2.61×10^{-3}	83.5
6	3.34×10^{-3}	88.6
7	4.18×10^{-3}	91.9
8	5.19×10^{-3}	95.9
9	5.90×10^{-3}	98.2
10	6.07×10^{-3}	99.1
11	6.15×10^{-3}	99.5

Appendix 2; Table 3. Wallace's (1948) data of Fort Peck Sand.

Readings	Coefficient of permeability	Degree of saturation
1	3.34×10^{-4}	88.6
2	3.34×10^{-4}	88.6
3	3.37×10^{-4}	88.6
4	3.45×10^{-4}	89.1
5	3.53×10^{-4}	89.8
6	3.69×10^{-4}	90.8
7	3.79×10^{-4}	91.8
8	3.95×10^{-4}	92.4
9	4.07×10^{-4}	93.6
10	4.17×10^{-4}	95.2
11	4.31×10^{-4}	96.0
12	4.73×10^{-4}	97.2
13	5.23×10^{-4}	99.6
14	5.23×10^{-4}	99.9
15	4.88×10^{-4}	98.1
16	5.00×10^{-4}	98.8

Appendix 2; Table 4. Wallace's (1948) data of Franklin Falls Sand.

Readings	Coefficient of permeability	Degree of saturation
1	1.84×10^{-2}	87.6
2	1.83×10^{-2}	87.8
3	1.87×10^{-2}	88.1
4	2.10×10^{-2}	89.7
5	2.18×10^{-2}	90.2
6	2.24×10^{-2}	90.5
7	2.38×10^{-2}	92.4
8	2.55×10^{-2}	94.1
9	2.81×10^{-2}	97.0
10	2.85×10^{-2}	97.4
11	2.87×10^{-2}	97.6
12	2.90×10^{-2}	97.6

Appendix 2; Table 5. Wallace's (1948) data of Ottawa Sand.

A P P E N D I X T H R E E

Appendix 3. The print out from MINITAB of the statistical analysis of Wallace's (1948) data for Union Falls A.

```
-- REGR C1  1 C2

                                                ST. DEV.      T-RATIO =
        COLUMN      COEFFICIENT                 OF COEF.      COEF/S.D.
        ---          -15.5078                    0.6584         -23.55
X1      C2            0.232826                  0.007084         32.87

THE ST. DEV. OF Y ABOUT REGRESSION LINE IS
S = 0.1163
WITH (    9-  2) =     7 DEGREES OF FREEDOM

R-SQUARED = 99.4 PERCENT
R-SQUARED = 99.3 PERCENT, ADJUSTED FOR D.F.

ANALYSIS OF VARIANCE

 DUE TO      DF          SS          MS=SS/DF
REGRESSION    1      14.61210        14.61210
RESIDUAL      7       0.09469         0.01353
TOTAL         8      14.70679


DURBIN-WATSON STATISTIC =  .86

-- PRINT C1 C2
COLUMN         C1          C2
COUNT           9           9
ROW
  1       4.63000      86.000
  2       4.69000      86.300
  3       4.76000      87.100
  4       5.22000      89.700
  5       5.76000      92.000
  6       6.78000      96.000
  7       7.49000      98.400
  8       7.76000      99.500
  9       7.75000     100.000
```

Appendix 3. The print out from MINITAB of the statistical analysis of Wallace's (1948) data for Union Falls B.

```
-- PRINT C1 C2
COLUMN          C1          C2
COUNT           11          11
ROW
  1       3.56000      83.600
  2       3.61000      83.800
  3       3.64000      84.400
  4       3.79000      86.000
  5       4.27000      88.800
  6       4.74000      91.400
  7       5.18000      93.300
  8       6.14000      97.000
  9       6.60000      98.800
 10       6.73000     100.000
 11       6.80000     100.000


-- REGR C1 1 C2

                                                ST. DEV.     T-RATIO =
        COLUMN      COEFFICIENT                 OF COEF.     COEF/S.D.
        --           -13.4054                    0.6758       -19.84
X1      C2            0.201092                0.007363        27.31

THE ST. DEV. OF Y ABOUT REGRESSION LINE IS
S = 0.1546
WITH (  11- 2) =    9 DEGREES OF FREEDOM

R-SQUARED = 98.8 PERCENT
R-SQUARED = 98.7 PERCENT, ADJUSTED FOR D.F.

ANALYSIS OF VARIANCE

 DUE TO      DF           SS          MS=SS/DF
REGRESSION    1      17.82932        17.82932
RESIDUAL      9       0.21515         0.02391
TOTAL        10      18.04447


DURBIN-WATSON STATISTIC =  .58
```

Appendix 3. The print out from MINITAB of the statistical analysis of Wallace's (1948) data for Fort Peck sand.

```
-- PRINT C1 C2
COLUMN          C1          C2
COUNT           11          11
ROW
  1       2.23000     78.3000
  2       2.20000     78.4000
  3       2.22000     79.2000
  4       2.42000     81.9000
  5       2.61000     83.5000
  6       3.34000     88.6000
  7       4.18000     91.9000
  8       5.19000     95.9000
  9       5.90000     98.2000
 10       6.07000     99.1000
 11       6.15000     99.5000


-- REGR C1 1 C2

                                                ST. DEV.     T-RATIO =
        COLUMN      COEFFICIENT                 OF COEF.     COEF/S.D.
        --           -12.9989                    0.9779       -13.29
X1      C2            0.19035                   0.01099        17.32

THE ST. DEV. OF Y ABOUT REGRESSION LINE IS
S = 0.3020
WITH (  11- 2) =    9 DEGREES OF FREEDOM

R-SQUARED = 97.1 PERCENT
R-SQUARED = 96.8 PERCENT, ADJUSTED FOR D.F.


ANALYSIS OF VARIANCE

 DUE TO      DF            SS          MS=SS/DF
REGRESSION    1       27.34888         27.34888
RESIDUAL      9        0.82059          0.09118
TOTAL        10       28.16947.


DURBIN-WATSON STATISTIC =   .45
```

Appendix 3. The print out from MINITAB of the statistical analysis of Wallace's (1948) data for Franklin Falls.

```
-- PRINT C1 C2
COLUMN          C1          C2
COUNT           16          16
ROW
  1       3.34000     88.6000
  2       3.34000     88.6000
  3       3.37000     88.6000
  4       3.45000     89.1000
  5       3.53000     89.8000
  6       3.69000     90.8000
  7       3.79000     91.8000
  8       3.95000     92.4000
  9       4.07000     93.6000
 10       4.17000     95.2000
 11       4.31000     96.0000
 12       4.73000     97.2000
 13       5.23000     99.6000
 14       5.23000     99.9000
 15       4.88000     98.1000
 16       5.00000     98.8000


-- REGR C1 1 C2

                                                ST. DEV.     T-RATIO =
        COLUMN      COEFFICIENT                 OF COEF.     COEF/S.D.
        --           -11.0451                    0.5483       -20.14
X1      C2            0.162073                  0.005851       27.70

THE ST. DEV. OF Y ABOUT REGRESSION LINE IS
S = 0.09550
WITH (  16- 2) =  14 DEGREES OF FREEDOM

R-SQUARED = 98.2 PERCENT
R-SQUARED = 98.1 PERCENT, ADJUSTED FOR D.F.

ANALYSIS OF VARIANCE

 DUE TO      DF           SS          MS=SS/DF
REGRESSION    1      6.999108        6.999108
RESIDUAL     14      0.127687        0.009120
TOTAL        15      7.126795

             X1           Y     PRED. Y    ST.DEV.
ROW          C2          C1      VALUE    PRED. Y    RESIDUAL      ST.RES.
 10        95.2      4.1700     4.3842     0.0256     -0.2142       -2.33R
 11        96.0      4.3100     4.5139     0.0276     -0.2039       -2.23R

R DENOTES AN OBS. WITH A LARGE ST. RES.

DURBIN-WATSON STATISTIC =  .86
```

Appendix 3. The print out from MINITAB of the statistical analysis of Wallace's (1948) data for Ottawa sand.

```
-- PRINT C1 C2
COLUMN        C1          C2
COUNT         12          12
ROW
  1      1.84000     87.6000
  2      1.83000     87.8000
  3      1.87000     88.1000
  4      2.10000     89.7000
  5      2.18000     90.2000
  6      2.24000     90.5000
  7      2.38000     92.4000
  8      2.55000     94.1000
  9      2.81000     97.0000
 10      2.85000     97.4000
 11      2.87000     97.6000
 12      2.90000     97.6000


-- REGR C1 1 C2

                                          ST. DEV.     T-RATIO =
        COLUMN      COEFFICIENT         OF COEF.     COEF/S.D.
        --            -7.1447            0.2809        -25.44
X1      C2           0.102844          0.003034         33.90

THE ST. DEV. OF Y ABOUT REGRESSION LINE IS
S = 0.04086
WITH (  12- 2) =  10 DEGREES OF FREEDOM

R-SQUARED = 99.1 PERCENT
R-SQUARED = 99.1 PERCENT, ADJUSTED FOR D.F.

ANALYSIS OF VARIANCE

 DUE TO      DF           SS         MS=SS/DF
REGRESSION    1      1.919067         1.919067
RESIDUAL     10      0.016699         0.001670
TOTAL        11      1.935766

            X1           Y     PRED. Y    ST.DEV.
ROW         C2          C1      VALUE    PRED. Y    RESIDUAL      ST.RES.
  6       90.5      2.2400     2.1626     0.0133      0.0774        2.00R

R DENOTES AN OBS. WITH A LARGE ST. RES.

DURBIN-WATSON STATISTIC =  .74
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